

Instructions: Do all your work in the blue examination booklets. Write answers IN THE GIVEN ORDER, though you may work on them in any order. You may use one page of prepared notes. Show *ALL* your work. You will get *little* or *no* credit for an unexplained answer. The value of each question appears in parentheses, 85 points in all. You have 80 minutes.

1. Consider the linear system

$$A' = (A|\underline{b}) = \left(\begin{array}{cc|c} 1 & 3 & 4 \\ -2 & 4 & 2 \end{array} \right).$$

- (a) (9 pts) Find A^{-1} using Gauss-Jordan elimination with NO row interchanges. Then use it to find the solution to $A\underline{x} = \underline{b}$. Explain all your steps.
 - (b) (10 pts) Find the *LUP* factorization of A using Gaussian elimination with partial pivoting, then back-solving for the solution. Again, explain what you are doing.
 - (c) (7 pts) Repeat the factorization and solution but now using scaled-partial pivoting, and explaining your steps. Is this solution unique? [prove your assertion].
 - (d) (9 pts) In this part, $A' = (A|\underline{b})$ is now a system of n linear equations in n unknowns where the coefficients satisfy (i) $a_{ij} = 0$ if $i + j \leq n$ and (ii) $a_{ij} \neq 0$ if $i + j = n + 1$. Argue that the system has a unique solution. How much work (the number of * and / steps used) is needed to find it? Explain how you got your answer.
2. We seek the roots of $f(x) = e^{x-1} - x$.
- (a) (4 pts) Show that $w = 1$ is the only root of f .
 - (b) (6 pts) Will Newton's method converge if you start at a value P_0 that is close enough to 1? Explain. If "YES", at what rate?
 - (c) (5 pts) Can you use bisection to find the root? Explain.
 - (d) *(5 pts) Describe what will happen if you use FPI on $g(x) = e^{x-1}$?
 - (e) (10 pts) Get three regula-falsi approximations to the root of $f(x) = x^3 - 4$ starting with the interval $(a, b) = (1, 2)$ and then accelerate the last one.

(OVER)

3. If you have time, please give some constructive criticism of the course: what's good? what's bad, and should be discontinued? what could be added to make it more useful?