Let me know if you have questions or spot errors or misprints

- 1a) $I_2(t) = A + B + C$ where $A = f(1) * [(x-2)*(x-4)]/[(-1)*(-3)]$, $B = f(2)*[(x-1)*(x-4)]/[(1)*(-2)]$, and $C = f(4)*[(x-1)*(x-2)]/[(3)*(2)]$.
- 1b) I counted 2 +, 6 -, 3 * and 8 /.
- 1c) $a_0 + a_1 * x_i + a_2 * (x_i) * *2 = f(x_i)$, for $i = 0, 1, 2$. Plugging the $x_i's$ into the eqns we get: $1 \cdot a_0 + x_0 * a_1 + (x_0) * *2 * a_2 = f(x_0)(= 1/x_0]$ $1 \cdot a_0 + x_1 * a_1 + (x_1) * *2 * a_2 = f(x_1)(= 1/x_1]$ $1 \cdot a_0 + x_2 * a_1 + (x_2) * *2 * a_2 = f(x_2)(= 1/x_2]$ and the x_i's are known, so solve for the a_i's.
- 1d) We have $I_2(x) = f(x)$ at $x = x_0, x_1, x_2$. We add $x_3 \neq x_i, i = 0, 1, 2$, to get $I_3(x) = I_2(x) + c_3 * (x - x_0) * (x - x_1) * (x - x_2)$ and evaluating that at $x = x_3$ shows that $c_3 = [f(x_3) - I_2(x_3)/[(x - x_0) * (x - x_1) * (x - x_2)]$

  To compute c_3 I counted $5*$ ops. and 1/ ops. (to evaluate $I_2(x_3)$ and then to compute the denominator, above).
- 1e) $T_1(t) = f(u) + (t - u) * f'(u) = 1/u + (t - u)(-1/u^2) = 1/2 - (t - 2)/4$.
- 1f) $E(T_1(t)) = f(t) - T_1(t) = f''(\theta) * (t - u)^2/2 =$ for some θ between t and u. Now use $f''(\theta) = 2/(\theta)^3$ and use the θ between u and t with smallest absolute val.
- 1g) $u_j = \cos((2 * j + 1) * \pi)/(2 * (n + 1))), j = 0, 1, \ldots, n$. On $[-1, 1]$ we get $u_0 = \cos(\pi/6) = \sqrt(3)/2$, $u_1 = 0$, and $u_2 = -u_0$.
- 1h) $f(1) = 1$ and $f(4) = 1/4$ and the line L through these two points $[A = (1, 1)$ and $B = (4, 1/4)]$ has equation $y = 5/4 - x/4$ and L is above f on this interval. The maximum of $L(x) - f(x) = 5/4 - x/4 - 1/x$ occurs when $L'(x) = f'(x)$, i.e., when $x = 2$ and that max. difference is $1/4$, So if we subtract $1/8$ from $L(x)$ (giving $y = 9/8 - x$) this new line equi-oscillates w.r.t f so it is the linear minimax approx. on $[1, 4]$.
- 1i)
- 1j)
- 3) This question tests your geometric understanding of polynomial approximation methods. Very little computation should be needed. Consider the function

  $$f(x) = \frac{1}{1 + 9x^2}, \quad -1 \leq x \leq 1$$

  and the following four approximations (also on $[-1, 1]$): $g(x) = 1$, $h(x) = 11/20$, $p(x) = 2/11$, $q(x) = 1/10 + 18(x + 1)/100$.

  - 3b) g is T_1
  - 3c) p is C_1
  - 3d) h is minimax
  - 3e*) g is better because dist(f,g)=9/10 with minimax dist.