CS323 **Some Test 1 Solutions** **April 9, 2004**

- 1b) The chord method will NOT converge. $m = 10$ is positive but $f'(x) = -3x^2$ is not. If $P_0 > w$, $P_n \to \infty$ and if $P_0 < w$, $P_n \to -\infty$.
- 2d) Newtons method is FPI on $g(x) = x - (9 - x^3)/(-3x^2) = 2x/3 + 3/x^2$, which has a fixed point at $w = 9^{1/3}$. Since $g'(x) = 2/3 - 6/x^3$ and $g''(x) = 18/x^4$, $g'(w) = 0$ but $g''(w) \neq 0$, the convergence rate will be (exactly) quadratic, if FPI does converge.

  To study the convergence, note that g is a contraction when

  $$-1 < 2/3 - 6/x^3 < 1.$$

  The right hand inequality is true for all $x > 0$ (as $w > 0$ we ignore it), and the left hand one when $x^3 > 18/5$, so g is a contraction on the set $S = ((18/5)^{1/3}, \infty)$. Observe that (i) $2^3 > 18/5$, so $P_0 \in S$; (ii) that $9 > 18/5$, so $w \in S$; (iii) $x \in S$ implies $g(x) \in S$. This is true because $g(w)$ is the min of $g(x)$ for all $x > 0$, and $g(w) = w \in S$. All conditions for the contraction mapping principle are satisfied so YES, it converges when $P_0 = 2$.

  Since $P_0 \notin S$ we are justified to say "I dont know". However for every $0 < P_0 < w$, $P_1 > w$ so $P_1 \in S$. Newtons method will converge here for *any* $P_0 > 0$.
- 5) The costs are THE SAME: To compute AC we study the cost for the j^{th} column of the product. The first element takes j * OPS, the second takes $j - 1$, etc., so the column j cost is $j(j+1)/2$, and the cost for all of AC is $\sum_{j=1}^{n} j(j+1)/2 = n^3/6 + n^2/2 + n/3$.

  For the Gauss-Jordan to reduce A to I, we need to "zero-out" column j above the j^{th} row. Suppose we already did the first $j - 1$ columns. For row $i < j$ we compute the pivot value $c = a_{ij}/a_{jj}$ and subtract $c{*}\text{row}_j$ from row_i. To do the same operation on the right-hand term which began as I, we just write the value c in position row_i col_j. This pivot step uses $n - j$ multiplications. Since there are $j - 1$ rows, the cost of column_j is $(n - j + 1)(j - 1)$. To reduce A to diagonal, we did $\sum_{j=2}^{n}(n - j + 1)(j - 1) = (n^3 - n)/6$ OPS. These operations on A, when performed on I, simply wrote the pivot values. Now we divide each row by the diagonal term $a_{i,i}$. The cost is $n(n+1)/2$ which when added to $(n^3 - n)/6$ gives $n^3/6 + n^2/2 + n//3$.