CS323 **Test 1** **March 25, 2004**

Instructions: Please attend to the following:

- Do all your work in the blue examination booklets.
- Write answers IN THE GIVEN ORDER, though you may work on them in any order.
- You may use a page of prepared notes, but all work must be your own.
- Show *ALL* your work. You will get *little* or *no* credit for an unexplained answer.
- The value of each question appears in parentheses. Use this as a guide in allocating your time. An asterisk (*) denotes a more challenging one. There are 80 points and 10 * points. You have about 75 minutes.

1. You want to find $w = 9^{1/3}$ (the cube root of 9) using the function $f(x) = 9 - x^3$, $x > 0$.
   (a) (7 pts) The plan is to use regula-falsi starting with the interval $I_0 = (2, 3)$. Is this possible? Explain. If YES, do one regula-falsi step.
   (b) (8 pts) Now the plan is to use the chord method with $m = 10$ and $P_0 = 2$. Do one chord step. Will the chord method converge in this case?
   (c) (5 pts) Do one step of Newton's method starting with $P_0 = 1$.
   (d) (10 pts) Investigate the convergence of Newton's method on f, starting at $P_0 = 2$. What about $P_0 = 1/2$? [Hint: $18/5 < 9$]

2. (10 pts) $\sqrt{6}$, the square root of 6, is 2.44948974278318...
   (a) Let x denote $\sqrt{6}$ in a $k = 4$ digit computer that rounds? What is x? Explain.
   (b) Let y denote $(\sqrt{6})/1000$ as computed by a $k = 5$ digit computer that chops. What is y? Explain.
   (c) Show how a $k = 3$ digit computer (with rounding) calculates the expression $(100 * \sqrt{6}) - (99 * \sqrt{6})$. What is the relative error of the result? Explain.

(OVER)

3. (10 pts) You are solving a linear system with the following coefficient matrix.

$$\begin{pmatrix} 3 & 1 & -2 & 3 \\ 0 & 0 & 2 & -4 \\ 0 & 2 & -2 & 1 \\ 0 & -4 & 5 & 6 \end{pmatrix}.$$

What is the pivot row for <u>column 2</u> if

(a) you are doing regular Gaussian elimination, no pivoting strategy? Explain your answer.

(b) you are doing Gaussian elimination with partial pivoting? Explain.

(c) you are doing Gaussian elimination with scaled partial pivoting? Explain.

4. You are given the matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \text{and} \qquad C = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

and vectors

$$\underline{b} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \qquad \text{and} \qquad \underline{d} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

(a) (10 pts) Solve $A\underline{x} = \underline{b}$ and $A\underline{x} = \underline{d}$ in the most efficient way you know. Carefully describe the steps you are taking.

i. Give the name of the method you are using.

ii. How many * and / steps do you use?

(b) (7 pts) Find C^{-1} using Gauss-Jordan reduction, carefully showing and explaining *each* row operation. Now use it to solve the system $C\underline{x} = \underline{d}$.

(c) (8 pts) Obtain the LU factorization of C, carefully showing and explaining each row operation. You may use compact notation if you wish. Now use L and U to solve the system $C\underline{x} = \underline{d}$, carefully explaining your steps.

(d) (5 pts) The methods in b) and in c) both solve $C\underline{x} = \underline{d}$. Which is the better one, and WHY?

5. (*10 pts) You are given n by n matrices A and C, both upper-triangular. You want to know if $C = A^{-1}$ and are considering the following two possible methods: (i) compute the product AC and see if it is I_n, the identity; (ii) compute A^{-1} directly and see if you get C. In both cases you will avoid multiplication by known 0's (zeroes) and 1's (ones).

Based on computational cost, which method do you choose, and WHY? Explain, giving details.