**CS323** **HW3b corrected** **Dec. 10, 2019**

Hand in solutions to problens marked with a (*) in class next Wed.
The other questions are for your own practice and benefit, but wont be graded.
Question 6 is changed from the earlier version.

1. (*) Let $f(x) = x^2$ on $[0, 1]$. Set up, then solve, the normal equations for $P_1(x)$, the (continuous) least squares approximation to $f$ of degree = one, expandeded *in the monomial basis*. Then find $d_2(f, P_1) \equiv \int_0^1 (f(x) - P_1(x))^2 dx$, the error of the approximation.

2. $C_1(x)$, the linear Tchebycheff interpolation of $f$ is $C_1(x) = x - 1/8$. Compute $d_2(f, C_1)$. Also compute $d_\infty(f, C_1) \equiv \max(|f(x) - C_1(x)| \, 0 \le x \le 1)$ and $d_\infty(f, P_1)$. What do these calculations show?

3. Now repeat using the basis $\phi_0(x) = 1$ and $\phi_1(x) = 2x - 1$. Check that the two expressions for $P_1$ are equivalent. Finally, show how to use the answer in 1) to find $P_1$ in the second basis without setting up and solving normal equations.

4. Repeat for $g(x) = e^x$ on $[0, 1]$.

5. (*) Take $x_i = i/5$, $i = 0, \ldots, 5$. Set up and solve the normal equations to find the *discrete* least squares straight line approximation to $f(x) = x^2$ [from (1)], in the monomial basis and then in the basis used in problem 3.

6. (*) Find $\phi_0, \phi_1, \phi_2$ the first three orthogonal basis functions for the interval $[-2, 3]$. Then find the coefficient matrix of the normal equations to determine the FIRST (i.e., degree at most ONE, or LINEAR), continuous least squares approximation to $f$ on this interval in the monomial basis.

7. Let $f(x) = x^3$ on $[-2, 3]$. Using the previous results,

   (a) Find $P_2(x)$, the degree at most two (i.e., quadratic) continuous-least-squares approximation to $f$, expanded in the monomial basis.

   (b) Now show how to express $P_2$ in the orthogonal basis using only the basis functions from 5)

   (c) Finally check the answer in b) by by solving the normal equations for $P_2$, expanded in the orthogonal basis.