CS442 **HW3b=HW3-revised** **Dec. 10, 2019**

Problems should solved with carefully written solutions.

You MUST SUBMIT IN PERSON (so we can exchange a few words) AT MY OFFICE from 3:00PM - 6PM on Wed., Dec. 11, or from 12-4:30PM on Thurs. or Fri. (Dec. 12 or 13).

As usual, dont forget to write and sign a pledge that your writeup is completely your own work and to name those students with whom you discussed the problems (NO CREDIT OTHERWISE)!

1. Some random-graph exercises:

   (a) Write down the sample space of labeled 4 vertex graphs that have 3 (undirected) edges, each equally likely. What is the probability that such a graph is connected? Explain.

   (b) Now the experiment is to start with $V = \{1, 2, 3, 4\}$ and $E = \phi$. At each step an edge $e \notin E$ is chosen uniformly at random and added to E until the graph is connected and the experiment concludes. Write down the sample space and give the probability of each graph in it.

   (c) We "showed" in class that the threshold in $G_{n,m}$ for the appearance of degree $= 2$ vertices is at about $m = .58n^{.5}$ edges. Repeat that exercise but now derive the threshold for vertices of degree $= 3$, explaining the details underlying your derivation.

2. Hamiltonian paths:

   (a) Find a tournament of size n with exactly one Hamiltonian path.

   (b) We know there exist tournaments of size n with at least $f(n) = n!/2^{n-1}$ Ham. paths. For $n = 5$ and $n = 6$ construct tournaments with $g(n) > f(n)$ Ham. paths. What is the biggest $g(n)$ you can come up with (the BIGGER the BETTER)?

3. Consider K_4 (the complete graph with 4 vertices and edges between each pair) and K_5, the complete graph with 5 vertices.

   (a) Make a drawing to depict each of the graphs and give the crossing number of each **drawing**. Then discuss the crossing number of each graph, giving either the crossing number, or the best bound on it you can explain.

   (b) Repeat for K_6.

   (c) Repeat for $K_{2,3}$ the complete bipartite graph with two sets of vertices; set A has two vertices and B has three vertices (so 5 in all), and $K_{2,3}$ has edges from each vertex in A to each vertex in B, and no other.

   (d) Then repeat for $K_{3,3}$