CS323 **HW3 + Practice** **Dec. 3, 2019**

Hand in solutions to problens marked with a (*) in class next Wed., Dec. 11, 2019.
The others are for your own practice and benefit, but wont be graded.

1. (*) Let $f(x) = x^2$ on $[0, 1]$. Set up, then solve, the normal equations for $P_1(x)$, the (continuous) least squares approximation to f of degree = one, expandeded *in the monomial basis*. Then find $d_2(f, P_1) \equiv \int_0^1 (f(x) - P_1(x))^2 dx$, the error of the approximation.

2. $C_1(x)$, the linear Tchebycheff interpolation of f is $C_1(x) = x - 1/8$. Compute $d_2(f, C_1)$. Also compute $d_\infty(f, C_1) \equiv \max(|f(x) - C_1(x)|\, 0 \leq x \leq 1)$ and $d_\infty(f, P_1)$. What do these calculations show?

3. Now repeat using the basis $\phi_0(x) = 1$ and $\phi_1(x) = 2x - 1$. Check that the two expressions for P_1 are equivalent. Finally, show how to use the answer in 1) to find P_1 in the second basis <u>without</u> setting up and solving normal equations.

4. Repeat for $g(x) = e^x$ on $[0, 1]$.

5. (*) Take $x_i = i/5$, $i = 0, \ldots, 5$. Set up and solve the normal equations to find the *discrete* least squares straight line approximation to f from (1), in the monomial basis and then in the basis in 2).

6. (*) Find ϕ_0, ϕ_1, ϕ_2 the first three orthogonal basis functions for the interval $[-2, 3]$. Then find the coefficient matrix of the normal equations to determine the second (i.e., degree at most two, or quadratic), continuous least squares approximation to f on this interval in *both* monomial and orthogonal bases.

7. Let $f(x) = x^3$ on $[-2, 3]$. Using the previous results,
   (a) Find $P_2(x)$, the degree at most two (i.e., quadratic) continuous-least-squares approximation to f, expanded in the monomial basis.
   (b) Now show how to express P_2 in the orthogonal basis using only the basis functions from 5)
   (c) Finally check the answer in b) by by solving the normal equations for P_2, expanded in the orthogonal basis.