CS442 **HW2 - revised** **November 13, 2019**

Problems should solved and carefully written solutions handed in by Nov. 20, 2019, in class, or by Nov. 21, at my office hours.. As usual, its OK to discuss problems with others but the writeup and explanations should be your own, and reflect your own understanding.

1. Recall that coupon collecting problem has a box with n coupons, each a small piece of paper with a different, distinct integer from 1 to n written on it. The experiment is to randomly sample a coupon (WITH REPLACEMENT) until a certain goal is achieved. In this question, the goal is achieved the moment you have first observed ALL if the <u>odd-numbered</u> coupons.
   - If n=2k is even, What is the expected "wait" (number of coupons you need to draw before you have finally seen all the odd-numbered ones.
   - Repeat the above where NOW, you need to observe coupon 1 and then, on the very next draw, coupon 2. The first draw where you get a 2 that comes immediately after a 1 was drawn, the experiment concludes. Carefully explain your answer.

2. John von Neumann suggested the following method to generate a random bit using a biased coin whose probability of HEAD is $p < 1/2$ (the bit you generate must be ZERO or ONE with probability 1/2 EACH).
   - Toss the coin twice:
   - if TH, output the bit ZERO
   - if HT, output the bit ONE
   - if anything else, NO OUTPUT (and you must repeat the experiment until you can output a bit).

   (a) Explain why you think this works, or doesnt work.

   (b) What is the mean and variance of the number of tosses needed to generate one bit?

3. The following questions pertain to the min-cut problem and to Karger's simple min-cut algorithm using $n-2$ randomly chosen contractions on an unweighted graph $G = (V, E)$ with n vertices and m undirected edges.

   (a) Suppose $G = (V, E)$ is a "cycle" with n vertices $v_1, \ldots, v_n$ and n edges $e_1 = \overline{v_1 v_2}, \ldots e_{n-1} = \overline{v_{n-1} v_n}$, and $e_n = \overline{v_n v_1}$. Argue that EACH distinct pair of edges forms a different min-cut in the graph - removing a pair creates two disjoint sets of connected vertices. (What happens if you only remove one edge?)

   (b) Use the success probability for Karger's algorithm to "find" a min cut to assert that NO graph on n vertices can have more min cuts than the n-cycle.

MORE MAY COME... prob. less than epsilon (i.e., small))