CS442 **HW2** **November 13, 2019**

Problems should solved and carefully written solutions handed in by Nov. 20, 2019. As usual, its OK to discuss problems with others but the writeup and explanations should be your own, and reflect your own understanding.

1. Recall that coupon collecting problem has a box with n coupons, each a small piece of paper with a different, distinct integer from 1 to n written on it. The experiment is to randomly sample a coupon (WITH REPLACEMENT) until a certain goal is achieved. In this question, the goalis achieved the moment you have first observed ALL if the odd-numbered coupons.
   - If n=2k is even, What is the expected "wait" (number of coupons you need to draw before you have finally seen all the odd-numbered ones.
   - Repeat the above where NOW, you need to observe coupon 1 and then on the very next draw, coupon 2. Explain your answer.
2. John von Neumann suggested the following method to generate a random bit using a biased coin whose probability of HEAD is $p < 1/2$ (the bit you generate must be ZERO or ONE with probability 1/2 EACH).
   - Toss the coin twice:
   - if TH, output the bit ZERO
   - if HT, output the bit ONE
   - if anything else, NO OUTPUT (and you must repeat the experiment until you can output a bit).

   (a) Explain why you think this works, or doesnt work.

   (b) What is the mean and variance of the number of tosses needed to generate one bit?

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