CS323 **HW2** **Nov. 18,, 2019**

Due in class Mon. Nov.25, 2019.

1. Given the following linear system

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 1 & 2 & -3 \\ 3 & 0 & 6 \end{pmatrix}, \ \underline{b} = \begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix}.$$

   (a) Use Gaussian elimination - *no pivoting* - to find the LU factorization of A *in compact notation*, showing and explaining each step. Write down L and U and show that $A = LU$

   (b) Using the above factorization, solve $A\underline{x} = \underline{b}$ by forward substitution (solve $L\underline{y} = \underline{b}$ for $\underline{y}$, then backsolving the system $U\underline{x} = \underline{y}$ to obtain $\underline{x}$.

   (c) Repeat (a), now using partial pivoting to get L, U, and $\underline{p}$ such that $LU = A(\underline{p})$ (A with its rows given according to the permutation vector $\underline{p}$). Verify that this gives the same solution.

2. Consider $f(t) = \sqrt{t}$.

   (a) Find T_0 and T_1, the Taylor polynomials for f of degrees zero and one, expanded about $u = 16/9$.

   (b) What are the approximations of $\sqrt{2}$?

   (c) Use Taylor's theorem to express the error of these approximations at the point t.

   (d) Use these expressions to *bound* the errors when $t = 2$. What do you learn about $\sqrt{2}$ in the two cases, above?

   (e) Repeat the above for T_2, the quadratic Taylor approximation, expanded about 49/25.

   (f) Compute the second Taylor polynomial for $f(t) = sin(\pi t)$, expanded about $t = 1/2$. Graph f and T_2 on $[0, 1]$. What does this approximation say about $\sqrt{2}$ (use $sin(\pi/4) = \sqrt{2}/2$)?

3. Given $f(t) = sin(\pi t)$ and collocation points $x_0 = -1/6$, $x_1 = 1/2$, and $x_2 = 0$.

   (a) Find Lagrange's form of $I_0(t)$ the degree $= 0$ interpolation of f based on x_0, and $I_1(t)$, the linear interpolation based on x_0 and x_1. Graph f, I_0 and I_1 on $[0, 1]$.

   (b) Using $sin(\pi/4) = \sqrt{2}/2$, write down the approximations of $\sqrt{2}$ given by I_0 and I_1.

   (c) Use the error formula to express the error of I_1 at $t = 1/4$. Then show how to get bounds on the approximation of $\sqrt{2}$.

4. Consider $f(t) = \sqrt{t}$ and collocation points $x_0 = 1$, $x_1 = 16/9$, and $x_2 = 9/4$.

   (a) Set up, then solve the equations to find $I_1(t)$, the straight line interpolating f at x_0 and x_1 *in the standard form*. What is its approximation of $\sqrt{2}$? Check that this agrees with Lagranges form of I_1.

   (b) Use the error formula to express the error, $f(2) - I_1(2)$, of the above approximation. Now bound the error. What do these bounds say about $\sqrt{2}$?

   (c) Finally find $I_2(x)$, the degree 2 polynomial interpolating f at x_0, x_1, and x_2, in Newton's form.