CS 323 **HW1 (corrected)** **October 9, 2019**

Clearly written solutions are due by 2PM. Fri., Oct. 18, 2019.
One or two submission boxes will be left outside Hill 417

If more convenient, you may submit at recitation on Oct. 17.

- **You may discuss the problems with other students but you are expected to write up your solutions entirely ALONE, and without the help or input from anyone else. When you finish, please write clearly on the first page the following pledge: "This write-up is entirely my own work - done alone, without help or input from others - although I did discuss the problems with" - (and then list the colleagues with whom you interacted for this HW).**

1. We want to represent each of the following real numbers in a $k = 7$ digit computer: (i) 1.817120593 (close to $(6)^{1/3}$); (ii) 1.414213562 (close to $\sqrt{2}$); (iii) -.0013456; (iv) 101.266.

   (a) Write down the normalized floating point representation using chopping. Compute the error and the relative error.

   (b) Repeat using rounding. Which method is more accurate?

2. Suppose you were computing $(6)^{1/3}(\omega)\sqrt{2}$ on a 6-digit computer, where (ω) stands for one of the arithmetic operators $+, -, *$ or $/$. Use (i) and (ii) in problem 1 for the "true" values.

   (a) Show what the 6 digit computer would obtain in each of the 4 possibilities for ω using the chopping convention.

   (b) Compute the relative errors of the calculations in a).

   (c) Repeat a) and b) using the rounding convention. Is rounding better than chopping?

3. x' is an approximation of 40.214213562 whose relative error has absolute value less than $5 * 10^{-5}$. Describe the largest interval in which x' can lie.

4. a', b', and c' are positive integers obtained by chopping the fractional part from a, b, and c, respectively. Find in terms of a', b', and c', the smallest interval containing

   (a) $a + b$.

   (b) $a \times b$ ($\times$ denotes multiplication).

   (c) $(a/b) \times c$.

5. Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. Then compute the relative errors of the final results.

   (a) $\frac{3}{4} + \frac{1}{5}$

   (b) $(\frac{3}{4})(\frac{1}{5})$

   (c) $(\frac{1}{3} - \frac{3}{11})\frac{4}{5} + \frac{3}{20}$

   (d) $(\frac{1}{3} + \frac{3}{11}) + \frac{3}{20}$

   (e) Repeat (a) and (c), above, now using four digits. Does this give greater accuracy? Is a $k + 1$ digit computer always more accurate than a k digit one?

6. We want to find the maximum of the function $f(x) = e^{-x} - \cos x$, $x \geq 0$.

   (a) On the same graph sketch e^{-x} and $\cos x$, $x > 0$.

(b) Argue that f has a global maximum in the interval $(0, 2\pi)$. Is this an appropriate starting interval for the bisection method applied to solving $f'(x) = 0$? Carefully explain.

7. You want to find the root of $f(x) = x^3 + 2$.

   (a) Starting with the interval $(-2, -1)$ (is this appropriate?) do two steps of bisection.

   (b) How many steps are needed to guarantee that the error is less than .005?

   (c) Now do two regula-falsi steps, as in (a). Which seems better? Explain.

   (d) Do two Newton steps starting with P_0, the initial bisection from (a).

   (e) Do two Secant steps starting with P_0 and P_1 from (a).

   (f) DO three stepts of the chord method using $m = 5$.

   (g) Now use Aitkin's accelleration to accellerate the last two approximations of the chord method, above.