CS-442 **HW1** **October 6, 2019**

Solutions are due in class on Oct. 16.

Its OK to discuss problems with others but the writeup and explanations should be your own, and reflect your own understanding. Please write and sign the following pledge with your HW: "This writeup is completely my own work, although I did discuss the problems and solutions with...", and then list your collaborators.

1. Players A, B, C, and D toss a fair coin in order (i.e., first A then B, then C, then D, then A again, etc.) The first player to get a HEAD wins, and the game is over. What are their respective chances to win? Repeat, now with only three players who toss *a pair* of fair coins, the first to get something different than HH is the winner, and the game is over.

2. We do the balls and boxes experiment with r balls and n boxes (each ball, labeled with its number - chooses one of the n boxes (each labeled with ITS number) uniformly at random and is placed in that box. $n > 5$ and each box has capacity $> r$.

   Let N denote the number of empty boxes, Y the number of boxes with exactly one ball, and L_i denotes the load on (i.e., number of balls in) box i.

   (a) Find the probability that $L_1 = k$, $k = 0, 1, \dots, r$.

   (b) Find the expected value for each of these random variables, N, Y, L_1, L_2.

   (c) Are N and L_1 independent? Explain. Repeat for N and Y; for L_1 and Y; for L_1 and L_2.

   (d) If $r = n + 1$, find the probability that $N = 0$. If $r = n - 1$ find the probability that $N = 1$; that $N = 2$.

   (e) What is the probability that ALL $L_i = 0$, $i = 1, \dots, 5$?

3. 4 cards from a 52 card deck are randomly dealt to each of 13 distinguishable players.

   (a) Describe the sample space and write down its size.

   (b) What is the probability that one of the players got all the aces? What is the probability that NOBODY got all the aces?

   (c) What is the probability that each player has one card from each suit? (Here, and on following parts, carefully explain your counting.)

   (d) What is the probability that *one* player has one card from each suit but that nobody else has cards from *more* than one suit?

4. (*) Try to show that the expected number of comparisons in Floyd-Rivest needed to get a "good" interval $[L, R]$ is at at most $3 * n/2 + o(n)$ [and if you dont believe its true, say why].

5. (*) The final step of the Floyd-Rivest randomized selection algorithm is to sort the (*random*) set $S = \{a_i \in \mathcal{A} : L \leq a_i \leq R\}$ of inputs which lie between sampled items L and R, once that set has been verified as "good" (because it must contain the item from A of rank k). Show that $P(|S| > 4n^{5/6}) \to 0$ as $n \to \infty$. (hints: look at the item $\sigma \in \mathcal{A}$ of appropriate rank and study the probability that $L < \sigma$; look at the item $\tau \in \mathcal{A}$ of appropriate rank and study the probability that $R > \tau$).