Informative Traces of Bisection and Regula-Falsi

We run seven steps of bisection and regula-falsi on $f(x) = x^2 - 2$, both using starting interval $I_0 = (u_0, v_0) = (1, 2)$: P_n is the n^{th} approximation (midpoint for bisection); $e_n = P_n - \sqrt{2}$ is the *error* at step n; r is the ratio of the current and previous errors

BISECTION

n	u_n	v_v	P_n	e_n	$r = \frac{e_n}{e_{n-1}}$
0	1.00000000	2.00000000	1.50000000	0.08578644	0.08578644
1	1.00000000	1.50000000	1.25000000	-0.16421357	-1.91421359
2	1.25000000	1.50000000	1.37500000	-0.03921356	0.23879611
3	1.37500000	1.50000000	1.43750000	0.02328644	-0.59383635
4	1.37500000	1.43750000	1.40625000	-0.00796356	-0.34198285
5	1.40625000	1.43750000	1.42187500	0.00766144	-0.96206161
6	1.40625000	1.42187500	1.41406250	-0.00015106	-0.01971724
7	1.41406250	1.42187500	1.41796875	0.00375519	-24.85852448

REGULA-FALSI

n	u_n	v_v	P_n	e_n	$r = \frac{e_n}{e_{n-1}}$
0	1.00000000	2.00000000	1.33333333	-0.08088023	-0.08088023
1	1.33333333	2.00000000	1.40000000	-0.01421356	0.17573593
2	1.40000000	2.00000000	1.41176471	-0.00244886	0.17229013
3	1.41176471	2.00000000	1.41379310	-0.00042046	0.17169602
4	1.41379310	2.00000000	1.41414141	-0.00007215	0.17159401
5	1.41414141	2.00000000	1.41420118	-0.00001238	0.17157650
6	1.41420118	2.00000000	1.41421144	-0.00000212	0.17157350
7	1.41421144	2.00000000	1.41421320	-0.00000036	0.17157298

NOTE:

1. Bisection errors change sign. **As $P_n \to \sqrt{2}$, the values "jump" around the limit**.
2. Note $|e_7|$ is much bigger than $|e_6|$. **Though $P_n \to \sqrt{2}$, it does not do so monotonically**.
3. In regula-falsi $v_n = 2$ always (why?). Regula-falsi errors are ALL negative, and consistently decrease in size.
4. r seems to be getting constant. $|e_n/e_{n-1}| \to .1715...$ **means the convergence rate of R.F. is LINEAR**.