Problem 1. Sketch the graph of $f(x) = \dfrac{x}{x^2 - 9}$.

Problem 2. Suppose that $f(x)$ is a twice differentiable function satisfying: $f(0) = 1$; $f'(x) > 0$ for all $x \neq 0$; and $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$.

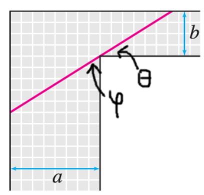
(a) Sketch a possible graph of $f(x)$.

(b) Let $g(x) = f(x^2)$. Prove that $g(x)$ has a unique local extremum at $x = 0$ and no points of inflection. Sketch a possible graph of $g(x)$.

.

Problem 3. Let $P = (a, b)$ lie in the first quadrant (so $a, b > 0$). Find the slope of the line through P such that the triangle bounded by this line and the axes in the first quadrant has minimal area. Then show that P is the midpoint of the hypotenuse of this triangle.

Problem 4. What is the longest pole that can fit around a corner between hallways of width a and b, as shown below?



Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

(a) From the picture, we can see that the length of the pole L is a sum of two hypotenuses. Using θ and φ as indicated above, give a formula for L in terms of this sum.

(b) Notice that θ and φ are complementary angles. Because of this, we can write a function $L(\theta)$ of the length of the pole dependent only on θ (and the constants a and b).

(c) The maximum L will be a be at a critical point of this function $L(\theta)$. Conclude that $L'(\theta) = 0$ under the condition that $\tan^3(\theta) = b/a$.

(d) Since $\tan\theta = \frac{b^{1/3}}{a^{1/3}}$, we can solve $\sin\theta$ and $\cos\theta$ for this particular angle. Putting everything together and simplifying, conclude that the maximum length is $L = (a^{2/3} + b^{2/3})^{3/2}$.