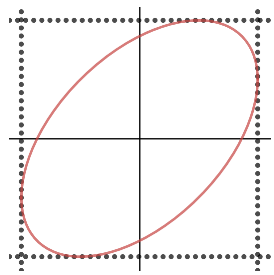
Problem 1. If a function is defined implicitly, there is no reason but habit that $\frac{dy}{dx}$ should be preferred over $\frac{dx}{dy}$. Consider the curve defined by the equation $y^3 + 1 = x^2 + y^2$.

(a) Compute both $\frac{dy}{dx}$ and $\frac{dx}{dy}$.

(b) Verify that the points $(1, 1)$ and $(7, 4)$ are both on the curve, then find the values of $\frac{dy}{dx}$ and $\frac{dx}{dy}$ at each.

(c) What do you notice about the relationship between these two versions of the derivative?

Problem 2. Consider the tilted ellipse $x^2 - xy + y^2 = 3$, shown below in a dashed box:



The box is tangent to the ellipse on each side. Using implicit differentiation, compute the equation of these four lines. Hint: if a horizontal line is when $\frac{dy}{dx} = 0$, what is a vertical line?

Problem 3. Calculate y'' at the point $(1, 1)$ on the curve $xy^2 + y - 2 = 0$ in these two steps:

(a) Compute $\frac{dy}{dx}$ symbolically and simplify it. Use it to get the value $y'(1, 1)$.

(b) Compute $\frac{d^2y}{dx^2}$ symbolically, then plug in $(x, y) = (1, 1)$ and $y'(1, 1)$ from above. You don't need to simplify the formula for y'' for this problem.

Problem 4. Consider $f(x) = \ln(x)$ and $g(x) = \ln(2x)$. Verify that $f'(x) = g'(x)$. Is there a simpler way to explain this?

Problem 5. Compute the derivative using the quotient rule (without simplifying all the way) of the function

$$h(x) = \frac{x(x+1)^3}{(3x-1)^2}$$

Repeat this process, but using logarithmic differentiation instead. Check that $h'(1)$ is the same for both methods.