Math 151:19-21 Workshop 5 **Ian Coley** **8 October 2019**

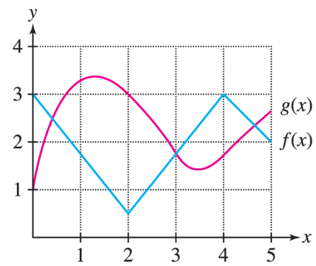
Problem 1. Students were asked to compute the derivative of $f(x) = \sin(e^x - x^2 + 2)$. Explain why each of the following answers is wrong, then compute the correct derivative.

(a) $f'(x) = \cos(e^x - x^2 + 2)$
(b) $f'(x) = \cos(e^x - 2x)$
(c) $f'(x) = \cos x \cdot (e^x - 2x)$
(d) $f'(x) = \cos(e^x - x^2 + 2) \cdot (e^x - x^2 + 2)$

Problem 2. Suppose that $f'(4) = g(4) = g'(4) = 1$. Do we have enough information to compute $F'(4)$, where $F(x) = f(g(x))$? If not, what is missing?

Problem 3. Using the quotient rule, verify that $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$. Then compute the second derivative of $\csc(x)$ (and simplify it as much as possible).

Problem 4. Consider the functions $f(x), g(x)$ pictured below:



Rogawski et al., *Calculus: Early Transcendentals*, 4e, © 2019 W. H. Freeman and Company

In case the colors fail, $g(x)$ is the wiggly one and $f(x)$ is the straight one. Estimate $(f \circ g)'(2)$ by visually inspecting the graph.

Problem 5. We said in class that sometimes higher derivatives give rise to a pattern. Let us examine the pattern in the following three cases:

(a) $\sin(x)$
(b) $\cos(2x)$
(c) $\frac{1}{x}$

Proceed using the following steps:

1. Compute the first, second, and third derivative of each.
2. Propose a general formula for $f^{(n)}(x)$, that is, the nth derivative. For (b) and (c), this should include n.
3. Compute the fourth derivative and see if it fits with your formula.

Here are a couple of useful facts. The product of the first n numbers is called a *factorial* and written $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$. A nice way to write $-, +, -, +, \ldots$ in an alternating way is $(-1)^n$, which is -1 when n is odd and $+1$ when n is even. Both of these tools will become extremely useful by the end of Math 152.