

Problem statement a) Suppose $f(x) = \frac{e^{10x}}{1 + e^{10x}}$. Graph this function when $-5 \leq x \leq 5$, and find the notable features of this graph, including any local extrema, points of inflection, and asymptotes. Sketch a plausible graph of $\frac{e^{10,000x}}{1 + e^{10,000x}}$

b) Suppose $g(x) = \frac{x^{10}}{1 + x^{10}}$. Graph this function for $-5 \leq x \leq 5$, and find the notable features of this graph, including any local extrema, points of inflection, and asymptotes. Sketch a plausible graph of $\frac{x^{10,000}}{1 + x^{10,000}}$

Note Such functions may serve as appropriate models for biophysical phenomena where rate constants in reactions are very different from everyday time scales. The curves sketched in a) are called *logistic curves*.

Problem statement $f(x) = \frac{e^x}{x^2 - 1}$

Give the domain of the function. Then, compute the first and second derivatives and give the intervals where the function is increasing, decreasing, concave up, concave down.

Find all transition points, and indicate whether they are local max, min or inflection points.

Indicate all the horizontal and vertical asymptotes if there are any. Then sketch the graph of the function.

Problem statement Consider the functions given by the equation $f_c(x) = (x^2 + c)e^x$, where c is a parameter.

a) Use the calculator to observe the curves for the values $c = 0, 1, 2$ when x is in the interval $[-4, 1]$. Do all three curves have the same number of horizontal tangents? Do all three curves have the same number of inflection points? You may have to *zoom* in to investigate this.

b) Use calculus to determine the location of all the inflection points of the graph of $y = f_c(x)$. Your answer may depend on c .

c) At what values of c does the number of inflection points change? What are the values of c (if any) for which there is exactly one inflection point?

Problem statement A child wants to build a tunnel using three equal boards, each 4 feet wide, one for the top and one for each side as shown. The two sides are to be tilted at equal angles θ to the floor. What is the maximum cross-sectional area A that can be achieved?

