

The final exam may verify your understanding and knowledge of anything from the syllabus. But you should start by making sure that you know the fundamentals below very well.

### Fundamental concepts

- function, domain, range [p4]
- increasing/decreasing/monotonic function [p6]
- one-to-one function, invertible function, the inverse function [p34, 35, 36]
- polynomial/rational function [p21]
- interval, closed interval, open interval [p2]
- limit [p111]
- continuity [p81]
- derivative (measures rate of change), differentiability [p121, 129, 150, equation (4) on p153]
- absolute minimum/maximum, local minimum/maximum [see separate definitions]
- antiderivative/indefinite integral [p275, 276]
- the definite integral [as a signed area p302] (measures net change p322)

### Fundamental theorems/facts

- the equation of a line [p16]
- the quadratic function [roots, sign, monotonicity intervals, concavity] [see also “the quadratic function (roots and sign)”]
- trigonometry [Sections 1.4, 1.5, Problem 6(c) on Workshop 1, the second page of the textbook (with formulas)]
- basic limit laws [p 77]
- basic laws of continuity [p 83], continuity of composite functions [Thm 5, p 85], continuity of the inverse function [Thm 4, p 85]
- Rational functions, trigonometric functions,  $x \rightarrow e^x$ ,  $x \rightarrow \ln x$ ,  $x \rightarrow x^a$  are continuous on their domains
- The range (=image) of a continuous function  $f: I \rightarrow \mathbb{R}$  defined on an interval  $I$  is an interval. [The Intermediate Value Thm]
- The range (=image) of a continuous function  $f: [a, b] \rightarrow \mathbb{R}$  defined on a closed bounded interval  $[a, b]$  is a closed bounded interval. [see Thm 1, p 216]
- The Squeeze Thm [Thm 1, p 96].
- $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  [p96-98]

- $\lim_{x \rightarrow \infty} x^n = \infty$  if  $n > 0$ ,  $\lim_{x \rightarrow \infty} x^{-n} = 0$  if  $n > 0$ <sup>1</sup>

- 

$$\lim_{x \rightarrow -\infty} x^n = \begin{cases} \infty, & \text{if } n \text{ is an even positive integer,} \\ -\infty, & \text{if } n \text{ is an odd positive integer,} \end{cases}$$

$$\lim_{x \rightarrow -\infty} x^{-n} = 0 \text{ if } n \text{ is a positive integer.}$$

- $\lim_{x \rightarrow x_0} \frac{1}{f(x)} = \infty$  if  $\lim_{x \rightarrow x_0} f(x) = 0$  and  $f(x) > 0$  for all  $x \neq x_0$  in some interval  $(x_0 - \epsilon, x_0 + \epsilon)$  around  $x_0$ ;

$\lim_{x \rightarrow x_0^+} \frac{1}{f(x)} = \infty$  if  $\lim_{x \rightarrow x_0^+} f(x) = 0$  and  $f(x) > 0$  for all  $x$  in some interval  $(x_0, x_0 + \epsilon)$ ,  $\epsilon > 0$ ;  
(and the analogous statement for the left-hand limit)

- $\lim_{x \rightarrow x_0} \frac{1}{f(x)} = -\infty$  if  $\lim_{x \rightarrow x_0} f(x) = 0$  and  $f(x) < 0$  for all  $x \neq x_0$  in some interval  $(x_0 - \epsilon, x_0 + \epsilon)$  around  $x_0$ ;

$\lim_{x \rightarrow x_0^-} \frac{1}{f(x)} = -\infty$  if  $\lim_{x \rightarrow x_0^-} f(x) = 0$  and  $f(x) < 0$  for all  $x$  in some interval  $(x_0 - \epsilon, x_0)$ ,  $\epsilon > 0$ ;  
(and the analogous statement for the right-hand limit)

- 

$$\lim_{x \rightarrow -\infty} e^x = 0, \quad \lim_{x \rightarrow \infty} e^x = \infty,$$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty, \quad \lim_{x \rightarrow \infty} \ln x = \infty$$

- FIRST AND SECOND LIST OF DERIVATIVES (see website)
- FIRST AND SECOND LIST OF INDEFINITE INTEGRALS (see website)
- the equation of the tangent line to the graph of a differentiable function [*Dfn*, p 121]
- linearity rules for differentiability/derivatives [p 132]; Linearity of the Indefinite/Definite Integrals [p 277, 303]
- differentiability implies continuity [p 136]
- The Chain Rule [p 169], The Substitution Method/Change of Variables Formula [p329, 331]
- product and quotient rules [p 143, 145], the derivative of the inverse function [p 178]
- approximating  $f(x)$  by its linearization [p 210]
- *Fermat's thm* on local extrema [p 218]
- *Rolle's thm* [p 220]
- *The Mean Value Thm* [p 226] and its consequences (*Corollary* on p 227 and *The Sign of the Derivative thm* on p 227)
- *First Derivative Test for Critical Points* regarding local extrema [p 229], *Second Derivative Test* for local extrema [p 237]

---

<sup>1</sup> This is a particular case of

$$\lim_{x \rightarrow x_0} \frac{1}{f(x)} = 0 \text{ if } \lim_{x \rightarrow x_0} f(x) = \infty \text{ (or } -\infty \text{).}$$

- *Test for Concavity/Inflection Points* [p 235]
- *L'Hôpital's Rules* [p 241, 244],  $\lim_{x \rightarrow \infty} \frac{e^x}{x^n} = \infty$  [p245]
- properties of the definite integral (the definite integral as signed area, additivity of adjacent intervals, comparison) [p 302, 304, 305]
- *The Fundamental Theorem of Calculus, Parts I and II* [p 310, 316]