Coresets Meet EDCS: Algorithms for Matching and Vertex Cover on Massive Graphs

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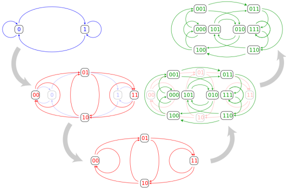
Joint work with MohammadHossein Bateni (Google), Aaron Bernstein (Rutgers), Vahab Mirrokni (Google), and Cliff Stein (Columbia)

Massive Graphs

Massive graphs abound in variety of applications: web graph, social networks, biological networks, etc.



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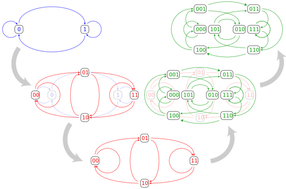
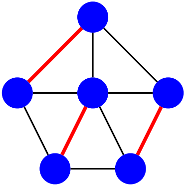


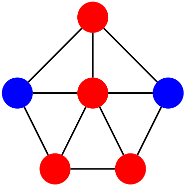
This talk: Matching and Vertex Cover problems on massive graphs.

Matchings and Vertex Covers

- Matching: A collection of vertex-disjoint edges.



- Vertex Cover: A collection of vertices containing at least one end point of every edge.



Matchings and Vertex Covers

Rich sources of inspiration for breakthrough ideas in computer science, algorithm design, and complexity theory.

Complexity class P

Approximation, parallel, online...

Hardness of approximation

Extended formulations

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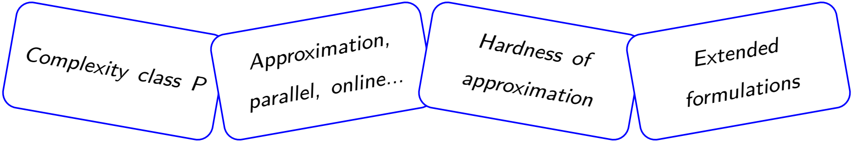

This talk:

Randomized composable coresets for matching and vertex cover.

Their applications to different models including streaming, distributed, and massively parallel computation.

Randomized Composable Coresets

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Algorithmic question. Design $\textnormal{ALG}$ with a good approximation ratio and a small size.

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Introduced first by [Mirrokni and Zadimoghaddam, 2015] for distributed submodular maximization.

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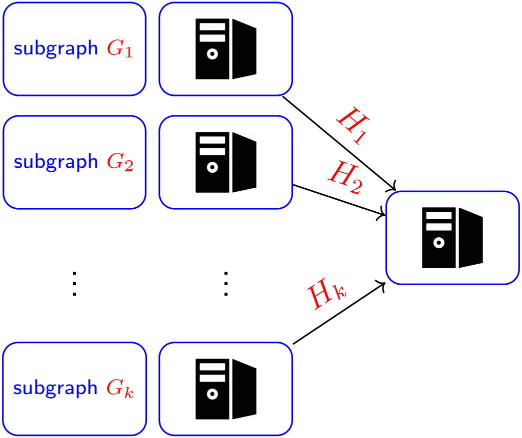
  - Direct applications to distributed communication, massively parallel computation, and streaming.

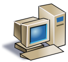
Randomized Composable Coresets: Applications

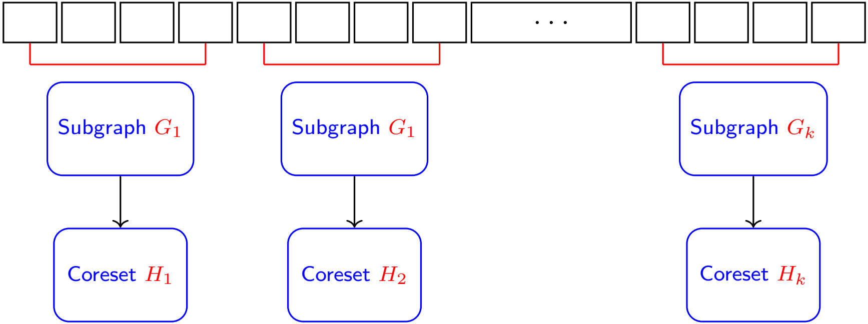
- An MPC algorithm with small memory per machine with one or two rounds of parallel computation.



Randomized Composable Coresets: Applications

- A streaming algorithm with small memory on random streams.





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    - $n^{o(1)}$-approximation requires $n^{2-o(1)}$ space.
  - Randomized composable coresets were suggested in [**A**, Khanna'17] to bypass these impossibility results.

State-of-the-Art

[**A**, Khanna'17]: There are $\widetilde{O}(n)$ size randomized composable coresets with:

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[**A**, Khanna'17] used this to obtain improved distributed and MPC algorithms.

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Questions.

- Improved randomized composable coresets?
- Compete with model-specific solutions using this general technique?

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We give significantly improved randomized composable coresets for matching and vertex cover.

Main Result. Randomized coresets of size $\widetilde{O}(n)$ with:

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Improve upon state-of-the-art in streaming, distributed, and MPC model in one or all parameters involved.

Direct Applications of Our Main Result

Corollary (Streaming)

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- [Konrad et al., 2012]: a 1.98-approximation to matching in random arrival streams with $\widetilde{O}(n)$ space.
- [Konrad, 2018]: improved approximation to 1.85 (following [Esfandiari et al., 2016, Kale and Tirodkar, 2017]).

Our Randomized Composable Coresets for Matching and Vertex Cover

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We mostly focus on maximum matching in this talk.

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We instead use edge degree constrained subgraphs to represent large matchings.

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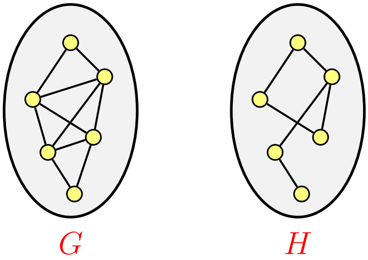

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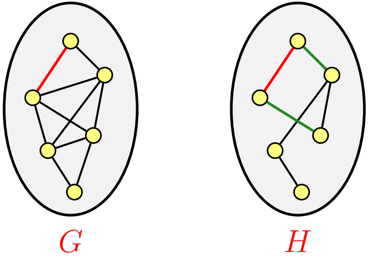

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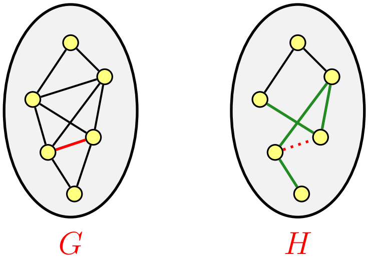

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Basic properties:

- A (β, ε)-EDCS has $O(n\beta)$ edges.
- Every graph admits a (β, ε)-EDCS for all $\varepsilon \in (0,1)$ and $\beta > 1/\varepsilon$.

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What is special about an EDCS for randomized composable coresets?

[this work]: W.h.p. on random partitions:

$$\text{EDCS}(G^{(1)}) \cup \ldots \cup \text{EDCS}(G^{(k)}) \approx \text{EDCS}(G^{(1)} \cup \ldots \cup G^{(k)}).$$

EDCS as a Randomized Coreset

Our main technical result:

Let $G^{(1)}, \ldots, G^{(k)}$ be a random partitioning of G.
Let $H^{(i)}$ be an arbitrary (β, ε)-EDCS of $G^{(i)}$.
Then $H^{(1)} \cup \ldots \cup H^{(k)}$ is a $\left(k\beta, \widetilde{\Theta}(\varepsilon)\right)$-EDCS of G w.h.p.

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Randomized Composable Coreset:

Let the randomized coreset be an arbitrary $\left(\widetilde{\Theta}(1), \widetilde{\Theta}(\varepsilon)\right)$-EDCS.

Size of each coreset is $\widetilde{O}(n)$.

Approximation follows from general properties of EDCS.

Proof Sketch of the Main Technical Result

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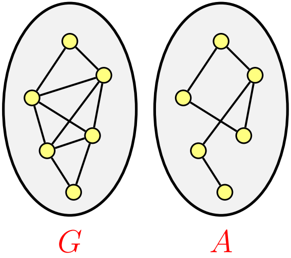
- Fix a $\left(k\beta, \widetilde{\Theta}(\varepsilon)\right)$-EDCS A of the input graph G.



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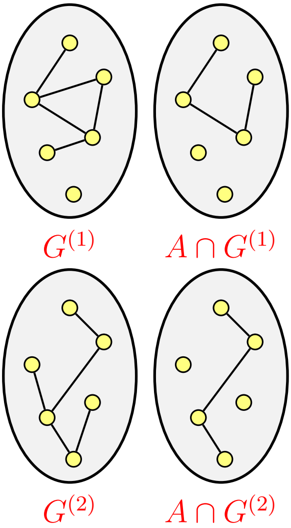

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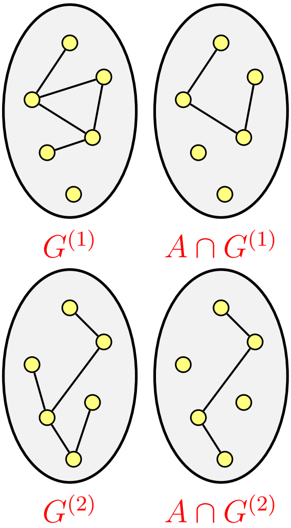

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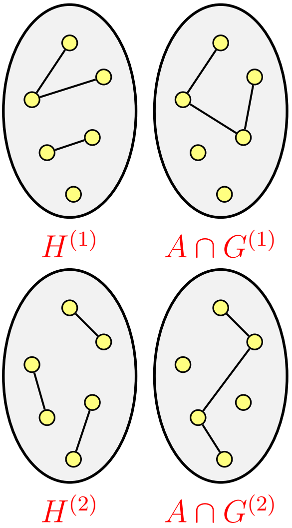

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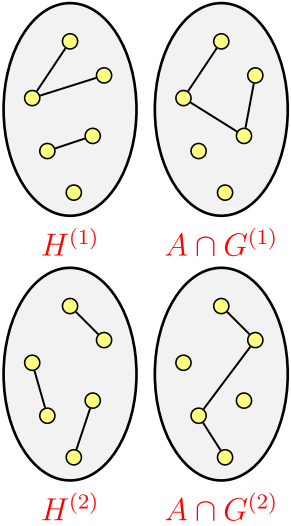

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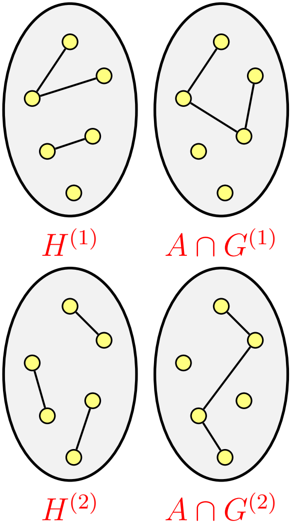

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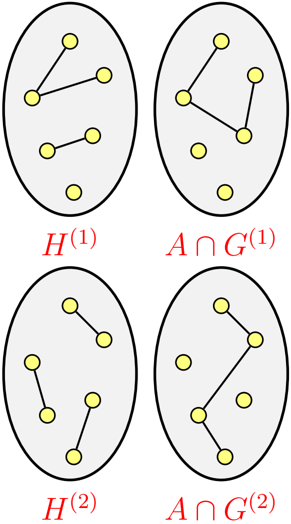

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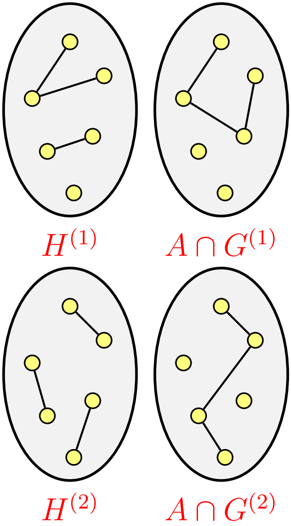

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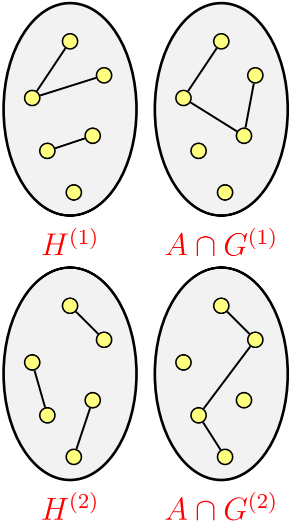
- Any fix?



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Let A and B be two (β, ε)-EDCS of a graph G. For all $v \in V(G)$:

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Proof Sketch of the Main Technical Result

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Enough to conclude that $H^{(1)} \cup \ldots \cup H^{(k)}$ is a $\left(k\beta, \widetilde{\Theta}(\varepsilon)\right)$-EDCS of G by the previous argument.

Wrap-Up

We proved,

Let $G^{(1)}, \ldots, G^{(k)}$ be a random partitioning of G.
Let $H^{(i)}$ be an arbitrary (β, ε)-EDCS of $G^{(i)}$.
Then $H^{(1)} \cup \ldots \cup H^{(k)}$ is a $\left(k\beta, \widetilde{\Theta}(\varepsilon)\right)$-EDCS of G w.h.p.

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Combined with general properties of EDCS, this implies:

Randomized composable coresets of size $\widetilde{O}(n)$ with:

- $(1.5 + \varepsilon)$-approximation for matching, and
- $(2 + \varepsilon)$-approximation for vertex cover.

Concluding Remarks

Distributed Sparsification

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We take this view to the next step for MPC algorithms.

Further Application to MPC

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To make this work:

- Vertex-based partitioning approach of [Czumaj et al., 2018].
- Additional care to not blow up approximation due to recursion.

Further Application to MPC

Corollary (MPC with low-memory per-machine)

An $O(\log\log n)$*-round MPC algorithm with* $O(1)$*-approximation to both matching and vertex cover and only* $O(n)$ *memory per-machine.*

Further Application to MPC

Corollary (MPC with low-memory per-machine)

An $O(\log\log n)$-round MPC algorithm with $O(1)$-approximation to both matching and vertex cover and only $O(n)$ memory per-machine.

- Can also give $(1+\varepsilon)$-approximation to maximum matching.
- Memory can be reduced to $O(n/\mathrm{polylog}(n))$.

Further Applications

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Subsequently,

- [Ghaffari et al., 2018]: $O(\log\log n)$ rounds; $(2+\varepsilon)$-approximation to both problems; $O(n)$ memory.

Concluding Remarks

Randomized composable coresets:

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Some key applications:

- A random arrival streaming $(1.5+\varepsilon)$-approximation to matching.
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Thank you!

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