CS 671: Graph Streaming Algorithms and Lower Bounds **Rutgers: Fall 2020**

Problem set 2

Due: 11:59PM, September 22, 2020

Please solve *exactly one* of the problems below.

Problem 1. Prove that for some $\varepsilon_0 := \Theta(n^{-o(1)})$, any one-way communication protocol for obtaining a $(1+\varepsilon_0)$-approximation to bipartite matching requires $\Omega(n^2)$ communication.

(This lower bound is weaker in terms of the approximation ratio but stronger in terms of the communication bound compared to what we proved in Lecture 2—you may want to prove this using a different construction of RS graphs mentioned in Lecture 2).

Problem 2. The goal of this problem is to show that EDCS also preserves a "small" vertex cover of G approximately (recall that a vertex cover is a set of vertices in the graph such that all edges are incident on at least one these vertices—the problem of finding a vertex cover of minimum size is intimately connected to maximum matching).

Let $G = (L, R, E)$ and H be a (β, ε)-EDCS of G for $\beta \geq 4/\varepsilon$. Define:

$$V_{high} := \{v \in L \cup R \mid \deg_H(v) \geq (1-\varepsilon) \cdot \beta/2\},$$
$$V_{vc} := \text{a minimum vertex cover of } H.$$

(*i*) Prove that $V_{high} \cup V_{vc}$ is a vertex cover of G (and not only H).

Hint: Use property (*ii*) of EDCS H to see how edges not in H can be covered.

(*ii*) Prove that $V_{high} \setminus V_{vc}$ has size at most $(1+2\varepsilon) \cdot |V_{vc}|$.

Hint: Consider the neighbors of $V_{high} \setminus V_{vc}$ in H; prove that (*i*) they should be part of V_{vc}, and (*ii*) their numbers cannot be much smaller than $V_{high} \setminus V_{vc}$.

Use the above to conclude that given only a $(\beta, \Theta(\varepsilon))$-EDCS of a graph and not the entire graph, we can still recover a $(2+\varepsilon)$-approximation to minimum vertex cover of the graph from it.

Question? Can you recover a better approximation from the EDCS? Say, an almost $(3/2)$-approximation?