Lecture 11: Practice Problem

April 7, 2020

Problem. In Lecture 11, we mentioned the following result:

- **Palette Sparsification Theorem [1]:** Let $G = (V, E)$ be an n-vertex graph with maximum degree Δ. Suppose for every vertex $v \in V$, we *independently and uniformly at random* sample $O(\log n)$ colors $L(v)$ from $\{1, \ldots, \Delta + 1\}$. Then, with high probability, there is a proper coloring of G in which the color of every vertex v is chosen from $L(v)$.

Based on this theorem, we showed how to design a semi-streaming algorithm for $(\Delta + 1)$ coloring.

Let us now consider sublinear time algorithms from the first half of the course. Use the palette sparsification theorem to give an $\widetilde{O}(n^{3/2})$ *query* algorithm for $(\Delta+1)$ coloring problem in the general query model (defined in Lecture 2). Note that for this problem, we only focus on the *query complexity* of the algorithms and not their time complexity although that can also be bounded (see [1]).

Hint: Give an $\widetilde{O}(n^{3/2})$ query and time algorithm for finding the *conflict graph* defined in the context of the Palette Sparsification Theorem; then apply this theorem to finalize the proof.

References

[1] S. Assadi, Y. Chen, and S. Khanna. Sublinear algorithms for $(\Delta + 1)$ vertex coloring. In *Proceedings of the Thirtieth Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2019, San Diego, California, USA, January 6-9, 2019*, pages 767–786, 2019. 1