Dynamic Discourse Referents for Tense and Modals

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Abstract

In this paper, we argue that both modals and tense expressions are anaphoric, and can be handled by the same discourse mechanisms underlying the resolution of pronouns and other anaphoric devices. In support of this, we show that both modals and tense give rise to sloppy identity patterns. We explain this by assuming all anaphors, including tense and modals, can access *dynamic* discourse referents. By treating sloppy identity as a general feature of the resolution of anaphoric devices in discourse, we give a simple, unified treatment of the sloppy identity patterns observed with tense and modals.

Keywords: Dynamic semantics, ellipsis, anaphora, tense, modals

1 Introduction

Example (1) illustrates anaphora involving NP's, VP's (VP ellipsis), tense, and modals:

(1)a **NP:** John gave *a presentation*. People enjoyed *it*.
b **VP:** John *gave a presentation*. Harry did not *VPE*.
c **Tense:** John *Past* give a presentation, and he *Past* leave.
d **Modal:** John *might* give a presentation. He *would* use slides.

In each case, the anaphorically related elements are italicized. In (1a), *a presentation* is the antecedent for the pronoun *it*, and in (1b), *gave a presentation* is the antecedent for the VPE. Similarly, we claim that in (1c), the *Past* in the first clause is the antecedent for the *Past* in the second clause. This is why the time of leaving is linked to the time of giving the presentation. In (1d), we claim that the possibility described by *John might give a presentation* is the antecedent for the modal *would*. This is why the claim of John using slides is relativized to just those states of affairs in which he gives the presentation.

The analogy between tenses and pronouns is an old one [Reichenbach, 1947; Partee, 1973]; data like (1d), first discussed in [Roberts, 1986; Roberts, 1989], has led a number of researchers recently to draw a similar analogy between modals and pronouns [Kibble, 1994; Portner, 1994; Geurts, 1995]. Such approaches seem initially at odds with the rich

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range of subtle interpretations assigned to tenses and modals in discourse. Far from remote, such interpretations can be found in the ordinary talk in which people propose and evaluate plans, the focus of current computational investigation of dialogue. Yet for some researchers, these interpretations could only derive from powerful general principles of relevance or accommodation [Lascarides and Asher, 1991; Roberts, 1989]. We propose to derive many such interpretations as instances of *sloppy* anaphora, as already observed with NPs and VPs. On our view, such sloppy interpretations actually confirm the parallels between tense and modals and pronouns, and illustrate the potential power and attractiveness of combining simple theories of anaphora with simple theories of attentional state in computational systems.

The following is a general characterization of sloppy identity:[1]

(2) C1 $\ldots[_{XP} \ldots[_{YP}] \ldots] \ldots$C2 $\ldots[_{XP'}]$
(C1, C2: "controllers" of sloppy variable **YP**)

We have an antecedent of category XP containing a sloppy variable YP. The interpretation of YP switches from controller C1 to C2. The following are familiar examples of sloppy identity, with the antecedent in italics:

(3)a **[NP [NP]]** Smith spent *[[his] paycheck].* Jones saved it.
b **[VP [NP]]** Susan *[loves [her] cat].* Jane does too.

In (3a), the NP *his paycheck* is the antecedent for the pronoun *it.* The embedded NP *his* is sloppy, switching from *Smith* to *Jones.* In (3b), the VP *loves her cat* is the antecedent for the VPE. The embedded NP *her* is sloppy, switching from *Susan* to *Jane.* Now we give two examples of sloppy identity involving tense and modals:

(4)a **[VP [Tense]]** You Past *[think I [Past] be crazy].* You probably still Pres do VPE.
b **[Modal [NP]]** John would use slides *if [[he] had to give the presentation].* Bill would just use the chalkboard.

In (4a), the antecedent for VPE is *think I past be crazy.* Here, the embedded *past* tense is the sloppy variable. It is anaphorically linked to the matrix past tense; the time I was (thought to be) crazy is the same as the thinking time. At the ellipsis site, the tense associated with *be crazy* could be resolved in a *strict* or *sloppy* manner: on the strict reading, the tense is still associated with the past thinking time of the first sentence. On the sloppy reading, the tense switches to the present matrix tense of the second sentence.

In (4b), the antecedent for the modal *would* is the possibility evoked by *if he had to give the presentation.* Furthermore, *he* is linked to *John.* In the second sentence, the modal *would* is anaphorically linked to the same possibility: *if he had to give the presentation.* The most natural reading here is sloppy: *he* switches to *Bill.* On the other hand, a continuation like *Bill would assist him* shows that strict reference is also possible.

To account for these facts, we describe a semantic framework in which tense and modals anaphorically access dynamic discourse referents introduced in prior discourse. Our general characterization of sloppy identity, in which any antecedent with an embedded anaphor can have a sloppy interpretation, emerges naturally in this framework. We show how the

[1] This framework has been applied to NP anaphora and VP ellipsis in [Hardt, 1993; Hardt, 1996].

framework accounts for a broad pattern of sloppy identity involving tense and modals. This pattern encompasses the above facts and many others.

These facts have not, to our knowledge, been previously observed in the literature. While they receive a simple, natural account within our framework, we suggest that they pose problems for many other accounts of anaphora, ellipsis, and modal subordination.

2 Framework

To begin, we briefly outline a semantic framework, adapting the compositional discourse representation theory (CDRT) of Muskens [Muskens, 1995].[2] Muskens's proposal encodes dynamic meanings as terms in typed λ-calculus, complementing the familiar types $\mathbf{e}$ (individuals), $\boldsymbol{\tau}$ (times), $\boldsymbol{\epsilon}$ (eventualities) and $\mathbf{w}$ (possible worlds) with a new type $\mathbf{s}$ that represent states of a discourse model. Discourse markers are functions from stores to objects; these objects may have any type. In this paper we will consider simple discourse referents with values of type $\mathbf{e}$ (introduced by NPs), $\boldsymbol{\tau}$ (introduced by tense), $\boldsymbol{\epsilon}$ (introduced by VP), as well as $(\mathbf{wt})$ (introduced by mood). Later, we will also consider *dynamic* discourse referents for all of the same types, in order to handle sloppy identity.

Muskens introduces an axiomatic theory that describes a relationship of branching possibility among worlds and events. He has a single domain for all possible worlds, but an existence predicate, e **in** w, says that event or entity e exists in world w. (Times exist in all possible worlds.) Further axioms define a predicate $\mathbf{mk}$ true of discourse markers and uses $\mathbf{mk}$ to ensure that markers and stores behave as we would expect for variables and assignments. Given this ontology, the meaning of a sentence φ is described by a relation that holds between stores i and j just in case j is a store that might result from the interpretation of φ in store i. The contents of the store at any point determine the discourse markers that are accessible to be the referents of pronouns.

Muskens treats the usual syntax of DRT as syntactic sugar that abbreviates more elaborate terms of typed λ-calculus. We will adopt the same strategy here; thus, despite differences in some of the underlying forms, we will be able to provide familiar and succint meaning-representations for discourse.

These differences arise because modality requires some changes in the types of objects and in the definition of accessibility. As we saw earlier, on a number of empirical grounds, the behavior of modality parallels that of pronouns and tense. In particular, modal subordination leads us to regard the antecedents of conditionals as introducing possibilities, which not only the consequent, but also subsequent sentences, can go on to describe. Contexts must be rich enough to capture these possibilities, and accessibility of referents must be relativized to the possibility in which those referents are needed.

Modality requires a type-theoretic change because conditional sentences describe sets of circumstances. For example, (5) describes possible wolves that could come in:

[2] [Muskens, 1995] is a more detailed presentation in the spirit of [Muskens, 1996], which includes an ontology of times, worlds and eventualities as well as ordinary individuals. Other compositional presentations of DRT exist (including notably [Groenendijk and Stokhof, 1990; Dekker, 1993]) and could be adapted along similar lines to those presented here. We find Muskens's presentation particularly straightforward.

(5) If a wolf comes in, it will eat you. Then it will eat me.

There are many possible worlds under consideration here; the identity of the wolf will vary from world to world. CDRT leaves the sets implicit by using a quantifier as part of the meaning of *if p q*: $\lambda i \lambda j.\ i = j \wedge \forall k(pik \supset \exists h.\ qkh)$. The different possible wolves are described by the different alternative values of the store k. Now, however, the set of circumstances must remain available for future assertions and thus must be explicit.

Our resolution to this problem involves three steps. First, we have markers for sets of worlds. These will be the markers that modals introduce and refer to; because they are sets, they can faithfully represent the possibilities evoked by *if*-clauses as in (5). To establish such sets of worlds, we borrow from Lewis [Lewis, 1973] a ternary relation **closer** on worlds, such that $\mathbf{closer}(w, w', w'')$ holds just in case w' is more like w than w'' is. To describe the possibility *if* φ, we use this relation to obtain the set of worlds closest to the actual world where φ is true.[3] Second, different values of a discourse marker may now be needed across possible worlds, so all discourse markers take a world argument as well as a store argument. δiw picks out the entity associated with marker δ in store i and world w. World discourse markers are no exception: which worlds they lead to depends on the world where they start. We will frequently use expressions of the form $\forall ww'(w' \in \omega iw \supset Pw')$ to impose property P on all the worlds w' that a marker ω could describe in store i. (We let δ range over discourse markers (typically over events, entities and times) and ω range over world-set markers.) Finally, each atomic condition in a DRS is required to hold throughout a set of worlds; abbreviating such conditions is facilitated by the assumption that each primitive predicate and relation has a distinguished world argument.[4]

We recast Muskens's abbreviations to these assumptions, starting with formulas licensing changes in stores across worlds:

(6)
$$\begin{array}{ll} i[\omega : \delta_1 \ldots \delta_n]j & \forall \nu((\mathbf{mk}(\nu) \wedge \delta_1 \neq \nu \wedge \ldots \wedge \delta_n \neq \nu) \supset \forall w(\nu iw = \nu jw)) \wedge \\ & \forall ww'(w' \in \omega jw \supset (\delta_1 jw' \ \mathbf{in}\ w' \wedge \ldots \wedge \delta_n jw' \ \mathbf{in}\ w')) \\ i[\omega : \omega_2]j & \forall \nu((\mathbf{mk}(\nu) \wedge \omega_2 \neq \nu) \supset \forall w(\nu iw = \nu jw) \wedge \\ & \forall w(\neg \exists w'(w \in \omega jw') \supset \neg \exists w'(w' \in \omega_2 jw))) \end{array}$$

Like Muskens's original definitions, the conditions in (6) express that i and j differ at most in the values of $\delta_1, \ldots, \delta_n$ or ω_2. However, (6) also imposes constraints on the new values of these markers. An individual marker's value at a world reachable by ω must exist there, while a set-marker's value at a world must be empty unless reachable by ω.

We abbreviate atomic conditions as in (7); at each world reachable from ω, we test the relation or equality using the markers' value there for the current store:

(7)
$$\begin{array}{ll} R\{\omega, \delta_1, \ldots, \delta_n\} & \lambda i. \forall ww'(w' \in \omega iw \supset R(w', \delta_1 iw', \ldots, \delta_n iw')) \\ \delta_1\ \mathbf{is}_\omega\ \delta_2 & \lambda i. \forall ww'(w' \in \omega iw \supset \delta_1 iw' = \delta_2 iw') \end{array}$$

[3] A better alternative may be to follow [Kratzer, 1989] in using an ontology of *situations* rather than worlds, which (at the cost of a less familiar setup) may allow for more and smaller cases to be considered, and for a more precise statement of the truth-conditions of counterfactuals and other conditionals.

[4] Again, a number of other options have been proposed in the literature: [Kibble, 1994; Portner, 1994; Geurts, 1995; Kibble, 1995]; we adopt this one for simplicity and clarity. We find the others difficult to adapt to Muskens's framework; they all involve rather more complicated types and ontology (Portner's proposal cannot even be interpreted in classical set-theory).

The definitions in (8) follow Muskens closely:

(8) $[\omega : u_1 \ldots u_n | \gamma_1 \ldots \gamma_m]$ $\quad \lambda i \lambda j(i[\omega : u_1 \ldots u_n]j \wedge \gamma_1(j) \wedge \ldots \wedge \gamma_m(j))$
$K; K'$ $\quad \lambda i \lambda j \exists k(Kik \wedge K'kj)$

The treatment of modal notions, however, requires some new notation:

(9) $\mathbf{if}(\omega_1, \omega_2, K)$ $\quad \lambda ij.\exists k(i[\omega_1 : \omega_2]k \wedge Kkj) \wedge \forall hk(i[\omega_1 : \omega_2]k \wedge Kkh) \supset$
$\forall w w_1(w_1 \in \omega_1 iw \supset (\omega_2 h w_1 \subseteq \omega_2 j w_1 \vee$
$\exists w_2 w_3(w_2 \in \omega_2 j w_1 \wedge w_3 \in \omega_2 h w_1 \wedge \mathbf{closer}(w_1, w_2, w_3))))$
$\mathbf{not}(\omega_1, \omega_2)$ $\quad \lambda i.\forall w w_1(w_1 \in \omega_1 iw \supset \neg(w_1 \in \omega_2 i w_1))$
$\mathbf{may}(\omega_1, \omega_2)$ $\quad \lambda i.\exists w w_1(w_1 \in \omega_1 iw \wedge w_1 \in \omega_2 i w_1)$

Statements of the form $\mathbf{if}(\omega_1, \omega_2, K)$ are dynamic transitions with the same type as boxes. Their effect is threefold. The marker ω_2 is introduced into the discourse model, and thereby made available for reference inside the box K. Then we update by K. Finally, we ensure that any comparable values for ω_2 (obtained similarly) involve either a smaller or a more distant set of worlds. This makes ω_2 the set of closest K-worlds to ω_1.[5]

The conditions given by **may** and **not** in (9), meanwhile, are predicates of stores—the type of conditions in boxes. In contrast to ordinary dynamic theories, where such notions are captured by quantifying over stores, these defintions simply relate two sets of possible worlds. Given two markers, **may** says the second is possible from the first; **not** says the second is impossible from the first. We shall use these in conjunction with $\mathbf{if}(\omega_1, \omega_2, K)$ statements in intepreting natural language negation, *may* and *might*. For example, *not* will transform one sentence parameterized by times and worlds into another, as in (10).

(10) not^{ω_i} $\quad \lambda \mathcal{K} \lambda t \lambda \omega .\, \mathbf{if}(\omega, \omega_i, \mathcal{K} t \omega_i); [\,|\, \mathbf{not}(\omega, \omega_i)]$

Note that a counterfactual scenario is correctly made available for subsequent reference.

As presented in [Muskens, 1996], accessibility is now a weak notion, and does not even ensure the existence of the referent in the world where it is needed. We will therefore augment accessibility with an existence presupposition, which we formalize as follows. Existence of individual-marker δ throughout world-marker ω at a store i is represented as $E u \omega i \equiv \forall w w'(w \in \omega i w' \supset u i w \mathbf{\ in\ } w)$. To test this as a presupposition, we must make sure that it would have held in the current store no matter how, consistent with the prior discourse, that store was obtained. We use the simple definition of (11), modeled after Muskens's definition of accessibility, to capture this condition. $\mathbf{pre}(u, \varphi, j, \alpha)$ is true iff variable-occurrence u in φ satisfies presupposition α starting from state j. (We gloss over the intricacies pointed out in e.g., [Saeboe, 1996; van der Sandt, 1992; Geurts, 1995].)

(11) $\mathbf{pre}(u, \varphi, j, \alpha)$ $= \alpha(j)$, if φ is atomic
$\mathbf{pre}(u, \mathbf{if}(\omega, \omega_1, K), j, \alpha)$ $= \mathbf{pre}(u, K, j, \alpha)$, if u occurs in K
$\mathbf{pre}(u, [u_1 \ldots u_n | \gamma_1 \ldots \gamma_m], j, \alpha)$ $= \mathbf{pre}(u, \gamma_i, j, \alpha)$, if u occurs in γ_i
$\mathbf{pre}(u, K_1; K_2, j, \alpha)$ $= \mathbf{pre}(u, K_1, j, \alpha)$ if u occurs in K_1
$= \forall h(K_1 j h \supset \mathbf{pre}(u, K_2, h, \alpha))$, if u occurs in K_2

Thus u is ω-accessible in DRS γ if and only if $u \in \mathbf{acc}(u, \gamma)$ and $\forall i.\mathbf{pre}(u, \gamma, i, E u \omega)$.

[5] As Kibble [1995] points out, the condition defining ω_2 as *the* set, rather than *a* set, illustrates a general tendency of plural anaphors to refer to maximal sets (a phenomenon underscored in an E-type analysis).

Now we have a formal explanation of why discourses such as (12) are infelicitous, at least on the most natural interpretation:

(12) A wolf might come in. #It is hairy.

On that interpretation, we introduce a new world-marker with *might*, and introduce an individual marker relative to that world-marker to represent the possible wolf. The new world-marker need not include all the worlds compatible with reality (or what we know about it: remember how possibilities branch), so the new individual marker may have a nonexistent value in some of them. This is an unsuitable value for *it*, because the realistic verb *is* must refer to reality.

Let us consider in more detail the analysis of (5), repeated below:

(5) If a wolf comes in, it will eat you. Then it will eat me.

We shall treat (5) as arising from the indexed syntactic representation shown in (13):

(13) $[\,\mathrm{POS}_{\omega_0}\,[\,\mathrm{FUT}_{t_0}\,[\mathrm{if}^{\omega_1}\,[\,\mathrm{a}^{u_1}_{\omega_1,t_0}\,\mathrm{wolf}\,[\,\mathrm{come\ in}^{e_1,t_1}\,][\,\mathrm{FUT}_{t_1}\,[\,\mathrm{it}_{u_1}\,[\,\mathrm{eat}^{e_2,t_2}\,\mathrm{you}\,]]]]]]]\,;$
$[\,\mathrm{POS}_{\omega_1}\,[\,\mathrm{FUT}_{t_2}\,[\,\mathrm{it}_{u_1}\,[\,\mathrm{eat}^{e_3,t_3}\,\mathrm{me}\,]]]]$

The abstract operators, coindexing, and application highlight a number of assumptions we make about how the anaphoric dependencies of sentences factor into the dependencies of their constituent words and phrases.

First, sentence meanings are constructed as abstracts which return context changes only when provided a world marker and a time marker (type $(\boldsymbol{\tau}\mathbf{wsst})$). Anaphoric values for these parameters are determined as the final stage of semantic composition, by the action of abstract tense and modal operators (here FUT for future and POS for possible). The actual contribution of these operators is a presupposition about their referent, but for our purposes quite simple meanings suffice:

(14) POS_{ω_i} (or any modal) $\quad \lambda\mathcal{J}.\mathcal{J}\omega_i$
FUT_{t_i} (or any tense) $\quad \lambda\mathcal{K}\lambda\omega.\mathcal{K}t_i\omega$

In (13), the referents for the first sentence are a free modal marker ω_0 representing reality and a free temporal marker t_0; the referents for the second sentence are the modal marker ω_1 representing the possibility that a wolf comes in and the time marker t_2 representing the duration of the results of the possible wolf eating you. We adopt this system because it streamlines the treatment of nested *if* and *when* clauses. In (13), because *if* takes sentence abstracts as arguments, we can easily ensure that both antecedent and consequent describe the same modal marker ω_1, and that the sentence as a whole depends on appropriate mood and time markers. The meaning of *if* that does this is:

(15) $\mathrm{if}^{\omega_i} \quad \lambda\mathcal{K}\lambda\mathcal{J}\lambda t\lambda\omega.\,\mathbf{if}(\omega,\omega_i,\mathcal{K}t\omega_i)\,;\,\mathcal{J}\omega_i$

As this discussion of parameters anticipates, we assume that the tense of a telic VP introduces not only an event marker but also a marker for the duration of its result state, following [Webber, 1988]. Reference to this marker allows time to move forward in the discourse. Thus, the meanings for *come in* and *eat* are as in (16).

(16) $\mathrm{come\ in}^{e_i,t_j} \quad \lambda u\lambda t\lambda\omega.\,[\,\omega : e_i,t_j \mid \textit{come-in}\{\omega,t,e_i,u\},t_j\ \mathbf{is}_\omega\ \mathbf{result}(e_i)\,]$
$\mathrm{eat}^{e_i,t_j} \quad \lambda\mathcal{Q}\lambda u.\,\mathcal{Q}(\lambda v\lambda t\lambda\omega.\,[\,\omega : e_i,t_j \mid \textit{eat}\{\omega,t,e_i,u,v\},t_j\ \mathbf{is}_\omega\ \mathbf{result}(e_i)\,])$

While the temporal and modal interpretation of verbs and *if* and *when* clauses is

governed by top-level referents for the entire sentence, NP meanings refer directly to a time marker and to a world marker. For example, the meaning of *a* is given in (17).

(17) $\mathrm{a}^{u_i}_{\omega_j, t_k} \quad \lambda P \lambda Q \lambda t \lambda \omega .\, [\, \omega_j : u_i \mid]\, ;\; P u_i t_k \omega_j \, ;\; Q u_i t \omega$

As always, this is a function from an $\bar{\mathrm{N}}$ meaning, P, and a VP meaning, Q, to a sentence meaning; it introduces an individual marker u_i and ensures that both P and Q hold of u_i. Both P and Q depend on a world and time argument: for Q, the VP, they are passed on to top level; for P, however, they are fixed by reference as ω_j and t_k. This referential mechanism is consistent with observations of [Enç, 1986; Reinhart, 1995] and others about the relatively unconstrained temporal and modal scope of noun phrases.

The remaining meanings involved in the composition of (13) are unsurprising:

(18) wolf $\lambda u \lambda t \lambda \omega .\, [\mid \mathit{wolf}\{\omega, t, u\}]$
it$_{u_i}$ $\lambda Q \lambda t \lambda \omega .\, Q u_i t \omega$
you $\lambda Q \lambda t \lambda \omega .\, Q(\mathit{you}) t \omega$
me $\lambda Q \lambda t \lambda \omega .\, Q(\mathit{me}) t \omega$

The final translation of (5) in this system is given in (19):

(19) $\mathbf{if}(\omega_0, \omega_1, [\omega_1 : u_1, e_1, t_1 | \mathit{wolf}\{\omega_1, t_0, u_1\}, \mathit{come\text{-}in}\{\omega_1, t_0, e_1, u_1\}, t_1\ \mathbf{is}_{\omega_1}\ \mathbf{result}(e_1)]);$
$[e_2, t_2 | \mathit{eat}\{\omega_1, t_1, e_2, u_1, \mathit{you}\}, t_2\ \mathbf{is}_{\omega_1}\ \mathbf{result}(e_2)]; [e_3, t_3 | \mathit{eat}\{\omega_1, t_2, e_3, u_1, \mathit{me}\}]$

The reader can check that the meaning given in (19) is derivable from the meaning in (13) using the definitions in (15)–(18), the identities of the λ-calculus and Muskens's axioms on stores; and that referents satisfy accessibility and existence conditions when necessary.

3 Dynamic discourse referents and sloppy identity

[Hardt, 1996] proposes that sloppy identity involves references to *dynamic* discourse referents. Whereas an ordinary discourse marker has type $\mathbf{sw}\alpha$, a dynamic discourse referent is typelifted over parametrized context changes; it has type $\mathbf{sw}(((\mathbf{sw}\alpha)(\boldsymbol{\tau}\mathbf{wsst}))(\boldsymbol{\tau}\mathbf{wsst}))$. Dynamic discourse referents can record the *meaning* of a constituent; when that meaning is recovered, its interpretation can be sloppy because of the intervening context change.

We manipulate dynamic discourse referents by *assigning* meanings as their values, using a condition $\zeta \leftarrow f$, and by *applying* those values to arguments, using a parameterized box $\zeta \downarrow a$. These notions are defined thus:

(20) $\zeta \leftarrow f \quad \lambda i .\, \forall w\, (\zeta i w = f)$
$\zeta \downarrow a \quad \lambda t \lambda \omega \lambda i \lambda j . \exists w (\zeta i w a t \omega i j)$

(We use variables x_i to represent dynamic individuals, z_i for dynamic times, P_i for dynamic properties, ξ_i for dynamic sets of worlds, and ζ_i to schematize over any of these types.) As befits meanings, (20) ensures that the values of dynamic referents exist, and are in fact the same, in all possible worlds. Since this is the case, to use a dynamic marker we can pick its value at any possible world arbitrarily, and apply that to its argument.

We formalize the changes in context between uses of dynamic markers using the notion of *center-shift*. For each storage type $\mathbf{s}$ we assume a distinguished marker $\delta_{\alpha 0}$ that represents the discourse center. Expressions with a referent marked by * reassign the discourse center to point to that object. Reassignment is defined as in (21):

(21) $\delta_0 :=_\omega \delta_2 \quad \lambda i \lambda j (i[\omega : \delta_0]j \wedge (\delta_0 \ \mathbf{is}_\omega \ \delta_2)j)$

Expressions marked by $_*$ pick out the value of the discourse center. In the case of individuals, center-shift draws on the centering theory of [Grosz *et al.*, 1995]; in the case of times, center-reassignment corresponds to standard methods of updating the reference time of tense later in time as discourse progresses [Hinrichs, 1986; Partee, 1984; Muskens, 1995].

4 Tense and Sloppy Identity

We now examine examples of sloppy identity and tense, using the semantic framework we have developed. Consider the following example:

(22) You thought I was crazy. You still do.

We assign (22) the indexed syntactic representation shown in (23):

(23) $[\ \mathrm{REAL}_{\omega *}\ [\ \mathrm{PAST}_{t*}\ [\ \mathrm{you}\ [^{P_1}\ \mathrm{think}^{e_2,\omega_2}\ [\ \mathrm{PAST}_{t*}\ [\ \mathrm{I}\ [\ \mathrm{be\ crazy}^{e_3}\ ]]]]]]];$
$[\ \mathrm{REAL}_{\omega *}\ [\ \mathrm{PRES}^{t_4 *}\ [\ \mathrm{still}\ [\ \mathrm{you}\ [_{P_1}\ \mathrm{do}\ ]]]]$

As before, the indexing of this example requires some additional explanation. The second sentence of (22) exhibits VP ellipsis, which we interpret using dynamic property markers. The definitions in (24) dictate that a fully realized VP adds a new property to the context and an occurrence of VP ellipsis accesses a property from the context.

(24) $[^{P_i}{}_{VP}] \quad \lambda P \lambda u \lambda t \lambda \omega .\ [\ \omega : P_i \mid P_i \leftarrow P\ ]\ ; P_i \downarrow ut\omega$
$[_{P_i}\ VP] \quad \lambda u \lambda t \lambda \omega .\ P_i \downarrow ut\omega$

These definitions describe the subscripted and superscripted brackets in (23).

As stative predicates, *think* and *be crazy* neither introduce new time markers nor move discourse time forward [Hinrichs, 1986]. However, *think* does introduce a marker for the embedded context in which *I was crazy* is claimed to hold. Following [Portner, 1994], such markers account for free indirect discourse—main sentences taken as implicit descriptions of an individual's mental state.

Under these assumptions, the main VP of the first sentence means:

(25) $\lambda u \lambda t \lambda \omega .\ [\ \omega : P_1 \mid$
$P_1 \leftarrow \lambda u \lambda t \lambda \omega .\ [\ \omega : \omega_2, e_2, e_3 \mid \mathit{think}\{\omega, t, e_2, u, \omega_2\},\ \mathit{crazy}\{\omega_2, t_0, e_3, I\}\ ]\ ]\ ; P_1 \downarrow ut\omega$

Composition with subject, tense and modal operators imposes the dynamic property given by P_1 on *you* at t_0 and ω_0. This step actually introduces the ordinary discourse referents described by the utterance.

The next sentence receives the following interpretation:

(26) $[\ \omega_0 : t_4 \mid\ ]\ ;\ t_0 :=_{\omega_0} t_4\ ;\ [\ \mid \mathit{still}\{\omega_0, t_4\}\ ]\ ;\ P_1 \downarrow (\mathit{you})t_4\omega_0$

What does *this* occurrence of P_1 impose? Since time t_0 has shifted in value from its previous past value to the present time t_4, we get a sloppy tense reading for the elided VP. That is, the VP meaning P_1 represents the property of *thinking I be crazy at time* t_0—i.e., *thinking I* am *crazy.*

Note that a strict tense reading is also possible, if we do not have the center shift of the tense. This is achieved by indexing the second sentence as follows:

(27) $[\ \mathrm{REAL}_{\omega *}\ [\ \mathrm{PRES}^{t_4}\ [\ \mathrm{still}\ [\ \mathrm{you}\ [_{P_1}\ \mathrm{do}\ ]]]]$

On this indexing, its interpretation is just:

(28) $[\,\omega_0 : t_4 \mid\,]\,;\,[\,\mid \mathit{still}\{\omega_0, t_4\}\,]\,;\; P_1 \downarrow (\mathit{you})t_4\omega_0$

Because the center does not shift to t_4, the meaning of the elided VP, *thinking I be crazy at time* t_0, is equivalent to *thinking I* was *crazy*.

Where example (22) shows sloppy temporal reference within VP ellipsis, (29) shows sloppy temporal reference in the interpretation of a pronoun.[6]

(29) A woman over thirty-five has a better chance to marry today than she did in the 1950s.

The explicit noun phrase *a woman over thirty-five* contains a temporal reference whose antecedent is *today*, but *she* is interpreted as *a woman over thirty-five in the 1950s*. We can capture that schematically by the following indexing:

(30) A^2 woman over thirty-five$_*$ has a better chance to marry today^{1*} than she$_2$ did in the 1950s^{3*}.

Reference to a dynamic individual, together with temporal center-shift, from today to the 1950s, will explain the change in interpretation. As a naturally-occurring example, (29) inevitably contains features like genericity and comparison that make its complete explication beyond the scope of this presentation. Nevertheless, sloppy temporal reference, of the sort we predict, will have to be a part any such analysis.

5 Modals and Sloppy Identity

We now turn to sloppy modal discourse referents, as exemplified in (31):

(31) John will use slides if he gives the presentation. Bill will just use the chalkboard.

We assign an indexed structure to this example as follows:

(32) $[\,\mathrm{John}^{u_1*}_{\omega_1,t_0}\,[\,\mathrm{POS}_{\omega_1}\,[\,\mathrm{FUT}_{t_0}\,[\,\mathrm{if}^{\xi_2}\,[\,\mathrm{he}_{u*}\,[\,\text{presents}\,]][\,\mathrm{FUT}_{t_0}\,[\,\epsilon_{u*}\,[\,\text{use slides}\,]]]]]]]$
$[\,\mathrm{Bill}^{u_2*}_{\omega_1,t_0}\,[\,\mathrm{POS}_{\xi_2}\,[\,\mathrm{FUT}_{t_0}\,[\,\epsilon_{u*}\,[\,\text{just use chalkboard}\,]]]]]$

In these examples, we must assume *John* and *Bill* each raise to a position higher than the *if*-clause in order to account for the ability to refer to them using a pronoun inside the *if*-clause. With this assumption, the first sentence composes:

(33) $[\,\omega_1 : u_1 \mid \mathit{john}\{\omega_1, t_0, u_1\}\,]\,;\; u_0 :=_{\omega_1} u_1\;;\,[\,\omega_1 : \xi_2 \mid \xi_2 \leftarrow$
$\lambda\mathcal{J}\lambda t\lambda\omega.\mathbf{if}(\omega, \omega_2, [\,\mid \mathit{presents}\{\omega_2, t, u_0\}\,])\,;\,\mathcal{J}\omega\,]\,;\,\xi_2 \downarrow \lambda\omega.[\,\mid \mathit{use\text{-}slides}\{\omega, t_0, u_0\}\,]$

Evaluating the application of ξ_2 introduces the possibility that John gives the presentation, and asserts that John uses slides there. The second sentence composes thus:

(34) $[\,\omega_1 : u_2 \mid \mathit{bill}\{\omega_1, t_0, u_2\}\,]\,;\; u_0 :=_{\omega_1} u_2\;;\xi_2 \downarrow \lambda\omega.[\,\mid \mathit{use\text{-}chalkboard}\{\omega, t_0, u_0\}\,]$

Because of the individual-level center-shift, application of ξ_2 introduces the possibility that *Bill* gives the presentation, and asserts that Bill uses the chalkboard there.[7]

[6] This sentence is taken from Stephanie Coontz's book, *The Way We Never Were*.

[7] An anonymous reviewer draws our attention to the following variant of example (31):

(35) If a professor gives the presentation he will use slides. Bill will use the chalkboard.

6 Alternative Approaches

We have identified a large space of possible sloppy identity configurations, involving pairs of categories ranging over VP, NP, Tense, and Modal. We have described a uniform view of sloppy identity which accounts for the entire space of possibilities in terms of the general mechanisms for anaphora resolution. In this section, we examine alternative accounts of sloppy identity and modal subordination, in the light of the data presented here. While no other theories would account for the entire space of sloppy identity possibilities, we will examine some accounts which might be expected to cover certain parts of the space.

Consider first the theory of [Fiengo and May, 1994], which accounts for VP ellipsis in terms of a syntactic identity condition between the antecedent and the reconstructed elided material. This identity condition is insensitive to certain differences in feature values; for example, a pronoun can differ in number and gender, as in the following examples:

(36)a John fed his cat, and Susan did too. (fed *her* cat)
b John fed his cat, and the other boys did too. (fed *their* cats)

This difference in feature values under ellipsis is termed *vehicle change*. One might account for our *sloppy tense* examples as a reflection of vehicle change, where a verb differs in a tense feature. This account seems problematic, in that it does not treat tense as anaphoric, and thus the above effect is unrelated to the sloppy identity variation of pronouns under ellipsis. It appears, therefore, that this account would permit examples like (37):

(37) Harry thinks I am crazy, and Tom does, too. (think I *was* crazy)

But (37) cannot mean that. This follows from our account, since there is no PAST controller in the second sentence for the embedded verbal tense to switch to.

Another prominent account of sloppy identity in VP ellipsis is that of [Dalrymple *et al.*, 1991]. In this approach, higher-order matching is used to solve equations that represent ellipsis occurrences. Consider the following example:

(38)a Harry thought I was crazy. Tom did too.
b $P(Harry) = thought(Harry, crazy(I))$

The solution for P is $\lambda x.thought(x, crazy(I))$. This represents the semantic value of the elided VP. In setting up the equation, one must first determine the pairs of *parallel elements* in the antecedent and elliptical clause. In the above example, there is one such pair: <Harry, Tom>. [Dalrymple *et al.*, 1991] point out that *tense* can also be treated as a parallel element. This would permit an account of sloppy tense similar to that of our approach. Consider (4a), repeated as (39a):

(39)a You thought I PAST be crazy. You probably still PRES do.
b P(You,PAST) = think(You, PAST, crazy(I, PAST))

We feel the sloppy reading is somewhat degraded, although still possible (*Bill will use the chalkboard if Bill gives the presentation.*) One way to permit this reading in our approach would be to allow the NP "a professor" to refer to the center as a pronoun does, and not establish a new center. We leave this as an issue for further study.

In this case, we have two pairs of parallel elements: <you,you> and <PAST,PRES>. This would permit two solutions for P[8]:

(40)a $P_1 = \lambda$ x,Tns.think(x,Tns,crazy(I, Past)))
b $P_2 = \lambda$ x,Tns.think(x,Tns,crazy(I, Tns))

Applied to the parallel elements of the ellipsis clause, we would get these readings:

(41)a P_1 <You,PRES> = think(You,PRES,crazy(I, PAST)))
b P_2 <You,PRES> = think(You,PRES,crazy(I, PRES))

This corresponds to the strict and sloppy tense readings, respectively. By λ-abstracting over Tense as a parallel element in this way, we are in effect capturing the anaphoric connection involving Tense. Thus, the Equational framework permits an account similar to the one proposed in this paper. Of course, it should be emphasized that our account achieves this effect without the mechanism of higher order matching. Furthermore, it is not at all clear how similar sloppy identity effects not involving ellipsis would be captured, such as those involving modals.

We now turn to the approach of [Roberts, 1989] for modal subordination, as in (5):

(5) If a wolf comes in, it will eat you. Then it will eat me.

The first sentence is analyzed as the following DRS:

(42) $\Diamond\,[\,w \mid \mathit{wolf}(w),\ \mathit{enter}(w)\,]$

The second sentence is analyzed as a conditional involving a missing antecedent, which is recovered by copying the contents of the previous DRS, resulting in the following:

(43) $[[\,w \mid \mathit{wolf}(w), \mathit{enter}(w)] \Rightarrow [\,x, z \mid \mathit{you}(x), \mathit{eat}(z, x), z = w\,]]$

This approach differs from the current approach in that it uses a special copying mechanism to capture the relativized modal force in modal subordination, while in the current approach we appeal simply to the general mechanism for the recovery of anaphoric expressions. Furthermore, Roberts' mechanism of copying DRS's would not permit sloppy identity in modal subordination, as in example (4b), repeated as (44):

(44) John would use slides if he had to give the presentation. Bill would just use the chalkboard.

To account for this in Roberts' approach, it would be necessary to introduce some mechanism for reassignment of discourse markers in DRS's, such as our center shift mechanism. This would make it possible to handle sloppy identity in modal subordination, but it would be distinct from the mechanisms for other types of sloppy identity.

7 Conclusions

We have argued that tense and modality are anaphoric, and we have presented new data showing that they participate in the following general pattern for sloppy identity:

[8]The "primary occurrence constraint" discussed in [Dalrymple *et al.*, 1991] would presumably rule out two other potential solutions for P.

(45) C1 $\ldots[_{XP} \ldots[_{YP}] \ldots] \ldots$C2 $\ldots[_{XP'}]$
(C1, C2: "controllers" of sloppy variable **YP**)

We have shown that simple extensions to a dynamic semantics framework make it possible to give a uniform account of sloppy identity involving tense and modals.

Since XP and YP can range over NP, VP, Tense and Modal, this pattern gives rise to the following space of 16 possible patterns for sloppy identity:

(46)

[VP [NP]]	[NP [NP]]	[Tense [NP]]	[Modal [NP]]
[VP [VP]]	[NP [VP]]	[Tense [VP]]	[Modal [VP]]
[VP [Tense]]	[NP [Tense]]	[Tense [Tense]]	[Modal [Tense]]
[VP [Modal]]	[NP [Modal]]	[Tense [Modal]]	[Modal [Modal]]

Examples of the four cases involving VP's and NP's are given in [Hardt, 1996]. In this paper, we have presented new data in the categories [VP [Tense]], [NP [Tense]], and [Modal [NP]]. The following examples illustrate additional patterns in this space:

(47)a **[Modal [VP]]** Harry's vices can make him compliant—or belligerent. When Harry gambles, *if I asked him not to VPE*, he would stop. When he drinks, he'd only continue with renewed vigor.

b **[Modal [Tense]]** On Tuesdays, we respond quickly to problems. *If Bill came then*, I could answer him in a week. However, on Wednesdays, we are very slow, and I wouldn't have the answer for almost two weeks.

We expect that the remaining patterns in this space are also possible, although perhaps with some awkwardness. Alternative approaches are only able to deal with limited sections of this space – ellipsis theories like [Fiengo and May, 1994] and [Dalrymple *et al.*, 1991] would at most deal with the four cases with a VP containing another category, while modal subordination theories like [Roberts, 1989] might deal with cases where a modal antecedent contains another category. However, none of these accounts could provide the basis of a uniform account for the entire space. This is the primary virtue of our account.

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