Introduction to computation

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Today's schedule

- Computation
- Linear programming: a useful and generic technic to solve optimization problems
- Basic computational complexity theorem
  - how can we formally measure computational efficiency?
  - how can we say a problem is harder than another?

The last battle

	strength	minerals	gas	supply
Zealot 	1	100	0	2
Stalker 	2	125	50	2
Archon 	10	100	300	4

- Available resource:

mineral	gas	supply
2000	1500	30

- How to maximize the total strength of your troop?

Computing the optimal solution

	str	m	g	s
Z	1	100	0	2
S	2	125	50	2
A	10	100	300	4
	Resource	2000	1500	30

- Variables
  - x_Z: number of Zealots
  - x_S: number of Stalkers
  - x_A: number of Archons
- Objective: maximize total strength
- max $1x_Z + 2x_S + 10x_A$
- Constraints
  - mineral: $100x_Z + 125x_S + 100x_A \leq 2000$
  - gas: $0x_Z + 50x_S + 300x_A \leq 1500$
  - supply: $2x_Z + 2x_S + 4x_A \leq 30$
  - x_Z, x_S, $x_A \geq 0$, integers

Linear programming (LP)

- Given
  - Variables x: a row vector of m positive real numbers
  - Parameters (fixed)
    - c: a row vector of m real numbers
    - b: a column vector of n real numbers
    - A: an $n \times m$ real matrix
- Solve $\max \; cx^T$

  s.t. $Ax^T \leq b, \; x \geq 0$
- Solutions
  - x is a feasible solution, if it satisfies all constraints
  - x is an optimal solution, if it maximizes the objective function among all feasible solutions

General tricks

- Possibly negative variable x
  - $x = y - y'$
- Minimizing cx^T
  - max $-cx^T$
- Greater equals to $ax^T \geq b$
  - $-ax^T \leq -b$
- Equation $ax^T = b$
  - $ax^T \geq b$ and $ax^T \leq b$
- Strict inequality $ax^T < b$
  - no "theoretically perfect" solution
  - $ax^T \leq b - \varepsilon$

Integrality constraints

- Integer programming (IP): all variables are integers
- Mixed integer programming (MIP): some variables are integers

Efficient solvers

- LP: can be solved efficiently
  - if there are not too many variables and constraints
- IP/MIP: some instances might be hard to solve
  - practical solver: CPLEX free for academic use!

Q & A time