Economics and Computation
Homework 2

Please do not hand in. Just do it by yourself to prepare for the exam.

1 Highly Recommended

Problem 1. VCG mechanism. Alice and Bob are thinking about asking their boss for a coffee machine for their office. Suppose that Alice appreciates coffee more than Bob, i.e., she would have value \$40 for having the machine, and Bob's value is \$20. Purchasing the machine would result in a negative value for the Company of \$50. If no machine is set up, all have value \$0. Suppose a VGC mechanism is used to determine whether or not to set up the coffee machine and how much the agents A(lice), B(ob), and C(ompany) have to pay.

1. If all agents are truthful, what is the outcome of the mechanism, and the payment? Hint: there are two outcomes: purchase the machine and do not purchase the machine. Negative payment (potentially from the company) means transferring money to the company.

2. If Alice and Bob collude, how can they save money?

3. Even if Alice and Bob do not collude, why might this mechanism be problematic in practice?

Problem 2. Generalized 2nd price auctions. Recall that in an *ad auction* there are n bidders bidding for $m < n$ slots. Each bidder is interested in

getting only one slot. The slots are ranked from the top (first) to the bottom (last) on the right side of a webpage. The ith slot will get s_i clicks. We can assume that $s_1 > s_2 > \cdots > s_m$.

An outcome consists in two parts: an allocation of m slots to m different bidders, and the **pay-per-click payment** p_i for the ith slot. Each bidder j has a private value v_j^* for each **click**, thus her utility for getting the ith slot is $s_i(v_j^* - p_i)$.

Recall that the *generalized second price auctions (GSP)* is a popular mechanism for ad auctions currently used at many companies including Google. It works as follows.

- Rank the bids from high to low. Let $b'_1 > b'_2 > \cdots > b'_n$ denote these bids.
- Allocate the 1st slot to the bidder with the highest bid b'_1, the 2nd slot to the bidder with the second highest bid b'_2, ..., allocate the mth slot to the bidder with bid b'_m.
- For each $i \leq m$, let $p_i = b_{i+1}$.

Suppose $n = 4$ and $m = 3$; $s_1 = 100, s_2 = 60, s_3 = 40$; $v_1 = 10, v_2 = 9, v_3 = 7, v_4 = 1$. Show that GSP is not truthful in this case. That is, when all bidders report truthfully, at least one of the bidders have incentive to lie. Identify all bidders who have incentive to lie.

Problem 3. Consider the following setting: three medical students s_1, s_2, s_3 must be matched with three mentors m_1, m_2, m_3 in a hospital. Preferences of students and mentors are illustrated below:

s_1 : $m_1 \succ_{s_1} m_2 \succ_{s_1} m_3$ $\quad\quad$ m_1 : $s_3 \succ_{m_1} s_2 \succ_{m_1} s_1$

s_2 : $m_2 \succ_{s_2} m_1 \succ_{s_2} m_3$ $\quad\quad$ m_2 : $s_3 \succ_{m_2} \emptyset \succ_{m_2} s_1 \succ_{m_2} s_2$

s_3 : $m_1 \succ_{s_3} m_3 \succ_{s_3} m_2$ $\quad\quad$ m_3 : $s_1 \succ_{m_3} s_3 \succ_{m_3} s_2$

1. Using the Deferred Acceptance procedure, find a student-optimal matching and mentor-optimal matching. Show your steps.
2. Provide all stable matchings.

2 Food for Thought

Problem 4. Two bidders with independent private values compete for an item in a First Price Auction. Both bidders have value $v_j \sim Uniform[2,3]$. Show that $s(v_j) = \frac{v_j}{2} + 1$ for $j \in \{1,2\}$ is a BNE of the auction.

Problem 5. Suppose that there are two separate auctions on *eBay*: an auction for a *pair of skis* ends on Tuesday evening, and another auction for a *pair of ski poles* already ends Monday evening. Your value for the *pair of skis* alone is $v_{skis} = \$100.00$, for the poles alone you have value $v_{poles} = \$20.00$, but if you get both items, your value is $v_{skis+poles} = \$200.00$ for the package. Assume *eBay* would use a Second Price Auction.

1. Is it a dominant strategy to bid your true values for the individual items (v_{ski} and v_{poles}) in the two separate auctions? Explain.

2. If there are several bidders with different valuations, will truthfully bidding the individual values by all bidders leads to an efficient outcome? Explain.