CS 534: Computer Vision
Binary Image Analysis

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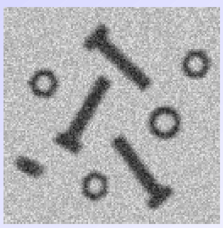
Outlines

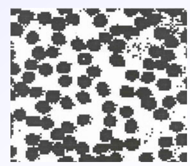
- A Simple Machine Vision System
- Image segmentation by thresholding
- Digital geometry
- Connected components
- Mathematical morphology
- Region descriptors
- Limitations

Problem definition

Objective: Design a vision system to “see” a “flat” world

- Page of text: text, line drawings, etc.
- Side panel of a truck
- Objects on an inspection belt
- X-ray image of separated potatoes
- Microscopic image of blood cells
- etc.





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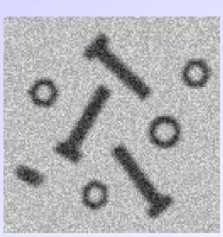
Problem definition

- General approach to recognition/inspection
  - Acquire gray scale image using camera
  - Reduce to black and white image - black objects on white background
  - Find individual black objects and measure their properties
  - Compare those properties to object models



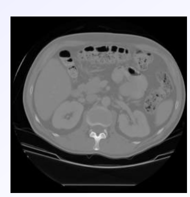


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Image segmentation

- How do we know which groups of pixels in a digital image correspond to the objects to be analyzed?
  - objects may be uniformly darker or brighter than the background against which they appear
    - black characters imaged against the white background of a page
    - bright, dense potatoes imaged against a background that is transparent to X-rays

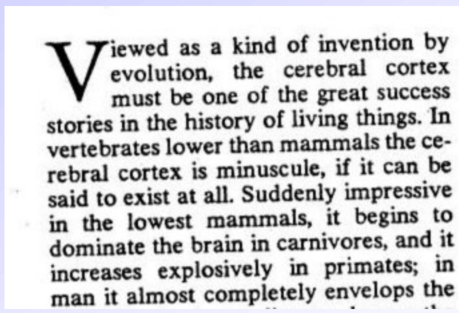
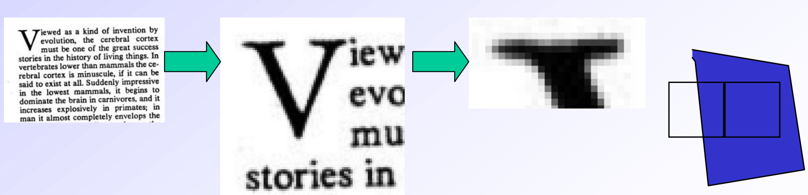

Viewed as a kind of invention by evolution, the cerebral cortex must be one of the great success stories in the history of living things. In vertebrates lower than mammals the cerebral cortex is minuscule, if it can be said to exist at all. Suddenly impressive in the lowest mammals, it begins to dominate the brain in carnivores, and it increases explosively in primates; in man it almost completely envelops the

Image segmentation

- Ideally, object pixels would be black (0 intensity) and background pixels white (maximum intensity)
- But this rarely happens
  - pixels overlap regions from both the object and the background, yielding intensities between pure black and white - edge blur
  - cameras introduce “noise” during imaging - measurement “noise”
  - potatoes have non-uniform “thickness”, giving variations in brightness in X-ray - model “noise”



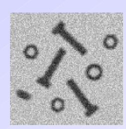
edge blur

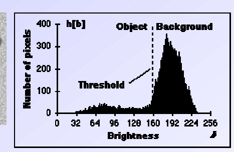
Image segmentation by thresholding

- But if the objects and background occupy different ranges of gray levels, we can "mark" the object pixels by a process called **thresholding:**
  - Let F(i,j) be the original, gray level image
  - B(i,j) is a binary image (pixels are either 0 or 1) created by thresholding F(i,j)
    - B(i,j) = 1 if F(i,j) < t
    - B(i,j) = 0 if F(i,j) >= t
    - We will assume that the 1's are the object pixels and the 0's are the background pixels

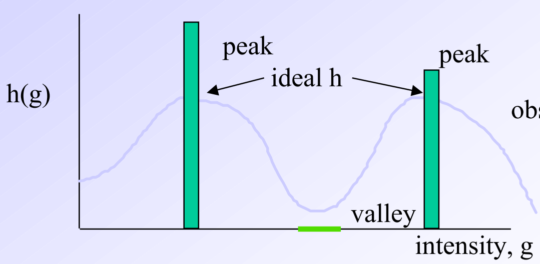
- How to choose the threshold t ?

Thresholding



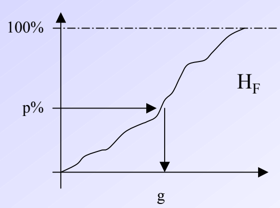


- How do we choose the threshold t?
- Histogram (h) - gray level frequency distribution of the gray level image F.
  - $h_F(g)$ = number of pixels in F whose gray level is g
  - $H_F(g)$ = number of pixels in F whose gray level is <=g (cumulative)



Thresholding

- P-tile method
  - in some applications we know approximately what percentage, p, of the pixels in the image come from objects
    - might have one potato in the image, or one character.
  - H_F can be used to find the gray level, g, such that ~p% of the pixels have intensity <= g
  - Then, we can examine h_F in the neighborhood of g to find a good threshold (low valley point)

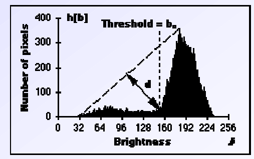


Thresholding

- Peak and valley method
  - Find the two most prominent peaks of h
    - g is a peak if $h_F(g) > h_F(g \pm \Delta g)$, $\Delta g = 1, ..., k$
  - Let g_1 and g_2 be the two highest peaks, with $g_1 < g_2$
  - Find the deepest valley, g, between g_1 and g_2
    - g is the valley if $h_F(g) <= h_F(g')$, g,g' in [g1, g2]
  - Use g as the threshold

Triangle algorithm

- A line is constructed between the maximum of the histogram at brightness b_{max} and the lowest value b_{min} = (p=0)% in the image.
- The distance d between the line and the histogram h[b] is computed for all values of b from $b = b_{min}$ to $b = b_{max}$.
- The brightness value b_o where the distance between $h[b_o]$ and the line is maximal is the threshold value.
- This technique is particularly effective when the object pixels produce a weak peak in the histogram.



Thresholding

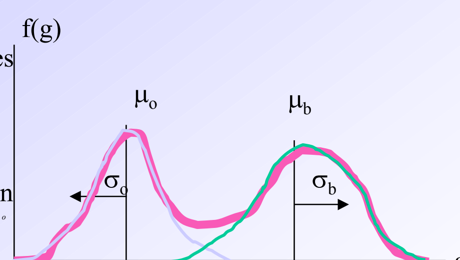
- Hand selection
  - select a threshold by hand at the beginning of the day
  - use that threshold all day long!
- Many threshold selection methods in the literature
  - Probabilistic methods
    - make parametric assumptions about object and background intensity distributions and then derive “optimal” thresholds
  - Structural methods
    - Evaluate a range of thresholds wrt properties of resulting binary images
      - one with straightest edges, maximum contrast, most easily recognized objects, etc.
  - Local thresholding
    - apply thresholding methods to image windows

An advanced probabilistic threshold selection method:
- minimizing Kullback information distance

- The observed histogram, f, is a mixture of the gray levels of the pixels from the object(s) and the pixels from the background
  - in an ideal world the histogram would contain just two spikes
  - but
    - measurement noise,
    - model noise (e.g., variations in ink density within a character) and
    - edge blur (misalignment of object boundaries with pixel boundaries and optical imperfections of camera)

  spread these spikes out into hills

- Make a parametric model of the shapes of the component histograms of the objects(s) and background
- Parametric model - the component histograms are assumed to be Gaussian
  - p_o and p_b are the proportions of the image that comprise the objects and background
  - μ_o and μ_b are the mean gray levels of the objects and background
  - σ_o and σ_b- are their standard deviations



$$f_o(g) = \frac{p_o}{\sqrt{2\pi}\sigma_o} e^{-1/2(\frac{g-\mu_o}{\sigma_o})^2}$$

$$f_b(g) = \frac{p_b}{\sqrt{2\pi}\sigma_b} e^{-1/2(\frac{g-u_b}{\sigma_b})^2}$$

- Now, if we hypothesize a threshold, t, then all of these unknown parameters can be approximated from the image histogram.
- Let f(g) be the observed and normalized histogram
  - f(g) = percentage of pixels from image having gray level g

$$p_o(t) = \sum_{g=0}^{t} f(g) \qquad p_b(t) = 1 - p_0(t)$$

$$\mu_o(t) = \sum_{g=0}^{t} f(g)g \qquad \mu_b(t) = \sum_{g=t+1}^{\max} f(g)g$$

- So, for any hypothesized t, we can "predict" what the total normalized image histogram **should** be if our model (mixture of two Gaussians) is correct.
  - $P_t(g) = p_o f_o(g) + p_b f_b(g)$
- The total normalized image histogram is **observed to be** f(g)
- So, the question reduces to:
  - determine a suitable way to measure the similarity of P and f
  - then search for the t that gives the highest similarity

Kullback information distance

- A suitable similarity measure is the Kullback directed divergence, defined as

$$K(t) = \sum_{g=0}^{\max} f(g)\log[\frac{f(g)}{P_t(g)}]$$

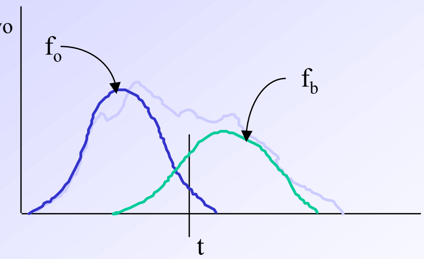
- If P_t matches f exactly, then each term of the sum is 0 and K(t) takes on its minimal value of 0
- Gray levels where P_t and f disagree are penalized by the log term, weighted by the importance of that gray level (f(g))

An alternative - minimize probability of error

- Using the same mixture model, we can search for the t that minimizes the predicted probability of error during thresholding
- Two types of errors
  - background points that are marked as object points. These are points from the background that are darker than the threshold
  - object points that are marked as background points. These are points from the object that are brighter than the threshold

An alternative - mimimize probability of error

- For each "reasonable" threshold
  - compute the parameters of the two Gaussians and the proportions
  - compute the two probability of errors
- Find the threshold that gives
  - minimal overall error
  - most equal errors

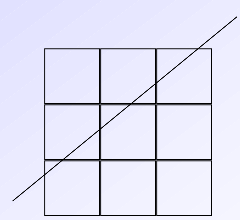


$$e_b(t) = p_b \sum_{g=0}^{t} f_b(g) \qquad e_o(t) = p_o \sum_{g=t+1}^{\max} f_o(g)$$

Elements of Digital Geometry

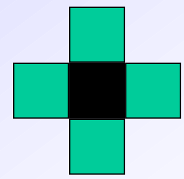
Digital Geometry:

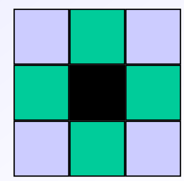
- Emerged with the rise of computer technologies in the second half of the 20th century
- Pioneers: A. Rosenfeld, J. L. Pfaltz
- Fundamental for computer graphics and digital image processing
- Deals with sets of grid points
- Mathematical roots: graph theory and discrete topology.
- Three Interrelated Areas:
  - Digital Topology: concepts of open and closed sets, connectedness, digital Jordan curve theorem, Euler Numbers,
  - Digital Geometry: the study of geometric properties of sets of lattice points produced by digitizing regions or curves in the plane: digitization, digital convexity, digital straightness.
  - Graph theory and combinatorics of digital spaces: regions on the digital grid as a undirected graphs



Elements of Digital Geometry - Connectivity

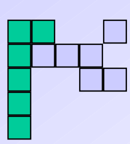
- Definition: Given a pixel (i,j) its 4-neighbors are the points (i',j') such that |i-i'| + |j-j'| = 1
  - the 4-neighbors are (i±i, j) and (i,j±1)
- Definition: Given a pixel (i,j) its 8-neighbors are the points (i',j') such that max(|i-i'|,|j-j'|) = 1
  - the 8- neighbors are (i, j±1), (i±1, j) and (i±1, j±1)

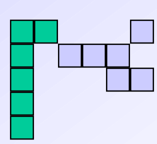




Adjacency

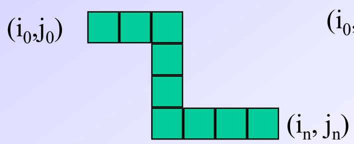
- Definition: Given two disjoint sets of pixels, A and B, A is 4-(8) adjacent to B is there is a pixel in A that is a 4-(8) neighbor of a pixel in B

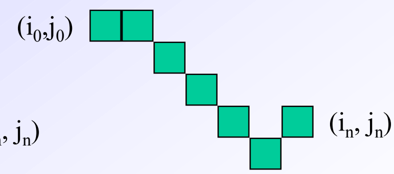




Connected components

- Definition: A 4-(8)path from pixel (i_0,j_0) to (i_n,j_n) is a sequence of pixels (i_0,j_0) (i_1,j_1) (i_2,j_2) , ... (i_n,j_n) such that (i_k, j_k) is a 4-(8) neighbor of (i_{k+1}, j_{k+1}), for k = 0, ..., n-1





Every 4-path is an 8-path!

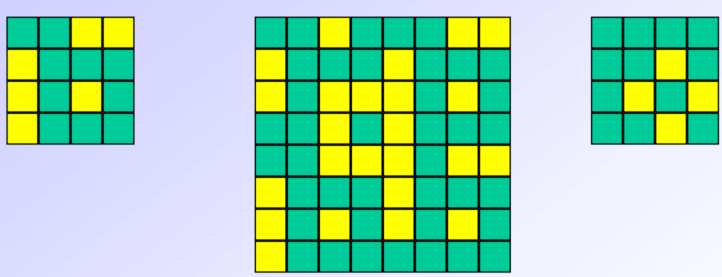
Connected components

- Definition: Given a binary image, B, the set of all 1's is called the foreground and is denoted by S
- Definition: Given a pixel p in S, p is 4-(8) connected to q in S if there is a path from p to q consisting only of points from S.
- The relation "is-connected-to" is an equivalence relation
  - Reflexive - p is connected to itself by a path of length 0
  - Symmetric - if p is connected to q, then q is connected to p by the reverse path
  - Transitive - if p is connected to q and q is connected to r, then p is connected to r by concatenation of the paths from p to q and q to r

Connected components

- Since the “is-connected-to” relation is an equivalence relation, it partitions the set S into a set of equivalence classes or components
  - these are called connected components
- Definition: S is the complement of S - it is the set of all pixels in B whose value is 0
  - S can also be partitioned into a set of connected components
  - Regard the image as being surrounded by a frame of 0’s
  - The component(s) of S that are adjacent to this frame is called the background of B.
  - All other components of S are called holes

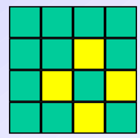
Examples - Yellow = 1, Green = 0



How many 4- (8) components of S?
What is the background?
Which are the 4- (8) holes?

Background and foreground connectivity

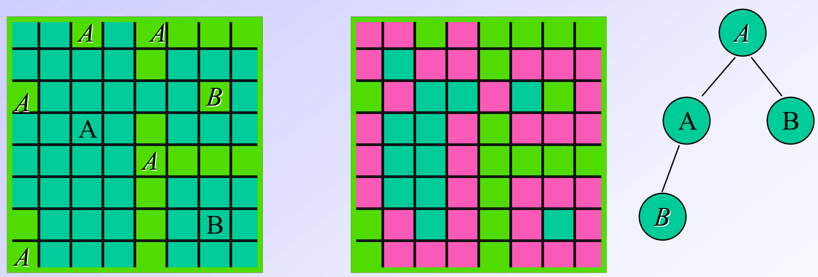
- Use opposite connectivity for the foreground and the background
  - 4-foreground, 8-background: 4 single pixel objects and no holes
  - 4-background, 8-foreground: one 4 pixel object containing a 1 pixel hole



Boundaries

- The boundary of S is the set of all pixels of S that have 4-neighbors in *S*. The boundary set is denoted as S'.
- The interior is the set of pixels of S that are not in its boundary: S-S'
- Definition: Region T surrounds region R (or R is inside T) if any 4-path from any point of R to the background intersects T
- Theorem: If R and T are two adjacent components, then either R surrounds T or T surrounds R.

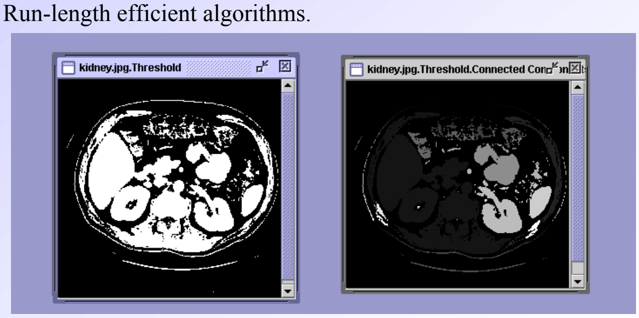
Examples



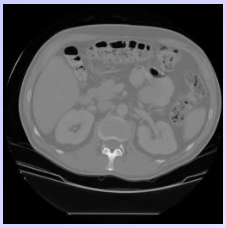
Even levels are components of 0's
The background is at level 0
Odd levels are components of 1's

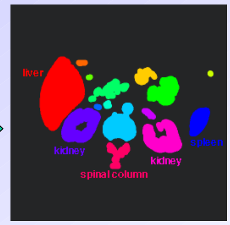
Component labeling

- Given: Binary image B
- Produce: An image in which all of the pixels in each connected component are given a unique label.
- Same as finding graph connected components
  - Recursive depth first labeling (almost never used)
  - Row-by-Row (most common) downward scan, upward scan O(n)
  - Run-length efficient algorithms.



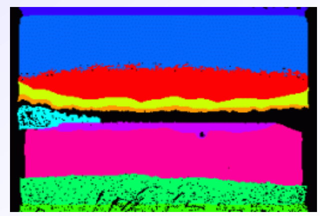
Labeling shown as Pseudo-Color





Connected components of 1's from thresholded image

Clustering –
Vector Quantization



Connected components of cluster labels

Mathematical Morphology

Binary mathematical morphology consists of two basic operations

dilation **and** **erosion**

and several composite relations

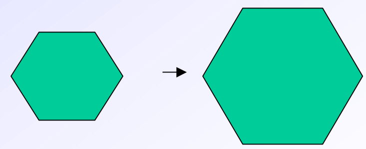
closing **and** **opening**
conditional dilation
. . .

Dilation

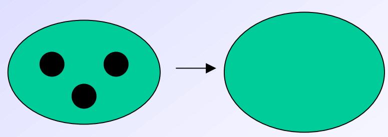
Dilation expands the connected sets of 1s of a binary image.

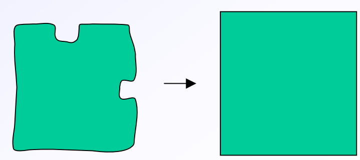
It can be used for

1. growing features



2. filling holes and gaps



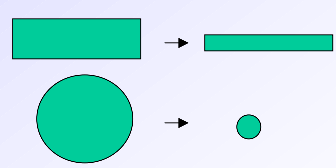


Erosion

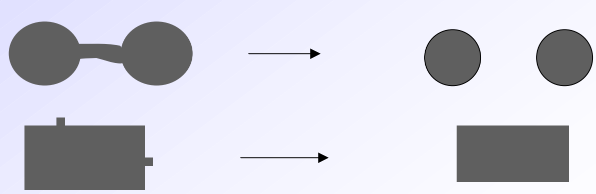
Dilation shrinks the connected sets of 1s of a binary image.

It can be used for

1. shrinking features



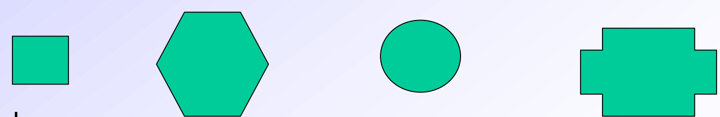
2. Removing bridges, branches and small protrusions



Structuring Elements

A structuring element is a shape mask used in the basic morphological operations.

They can be any shape and size that is digitally representable, and each has an origin.



box   hexagon   disk   something

box(length,width)   disk(diameter)

Dilation with Structuring Elements

The arguments to dilation and erosion are

1. **a binary image B**
2. **a structuring element S**

dilate(B,S) takes binary image B, places the origin of structuring element S over each 1-pixel, and ORs the structuring element S into the output image at the corresponding position.

$$\underbrace{\boxed{\begin{array}{cccc}0&0&0&0\\0&1&1&0\\0&0&0&0\end{array}}}_{B} \qquad \underbrace{\boxed{\begin{array}{cc}1&\\1&1\end{array}}}_{S} \xrightarrow{\text{dilate}} \underbrace{\boxed{\begin{array}{cccc}0&1&1&0\\0&1&1&1\\0&0&0&0\end{array}}}_{B \oplus S}$$

origin

Erosion with Structuring Elements

erode(B,S) takes a binary image B, places the origin of structuring element S over every pixel position, and ORs a binary 1 into that position of the output image only if every position of S (with a 1) covers a 1 in B.

origin

$$\underset{B}{\begin{bmatrix} 0&0&1&1&0\\ 0&0&1&1&0\\ 0&0&1&1&0\\ 1&1&1&1&1 \end{bmatrix}} \quad \underset{S}{\begin{bmatrix} 1\\ 1\\ 1 \end{bmatrix}} \xrightarrow{\text{erode}} \underset{B \ominus S}{\begin{bmatrix} 0&0&0&0&0\\ 0&0&1&1&0\\ 0&0&1&1&0\\ 0&0&0&0&0 \end{bmatrix}}$$

Example 1 to Try

$$B \quad \begin{bmatrix} 0&0&1&0&0&1&0&0\\ 0&0&1&1&1&1&1&0\\ 1&1&1&1&1&1&0&0\\ 1&1&1&1&1&1&1&1\\ 0&0&1&1&1&1&0&0\\ 0&0&1&1&1&1&0&0\\ 0&0&1&1&1&1&0&0 \end{bmatrix} \quad \overset{S}{\begin{bmatrix} 1&1&1\\ 1&1&1\\ 1&1&1 \end{bmatrix}} \xrightarrow{\text{erode}}$$

dilate with same structuring element

Example 2 to Try

B

0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

S

0	1	1	1	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	1	1	1	0

First erode and then dilate with the same S.

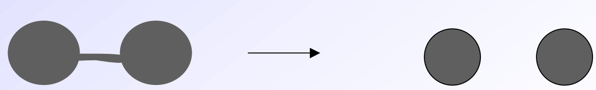
Opening and Closing

- Closing is the compound operation of dilation followed by erosion (with the same structuring element)

$$B \bullet S = (B \oplus S) - S$$

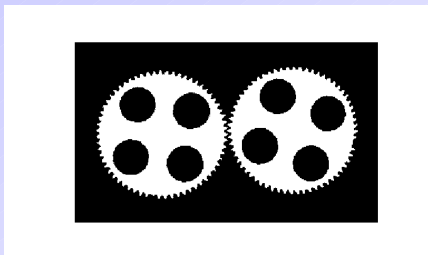
- Opening is the compound operation of erosion followed by dilation (with the same structuring element)

$$B \circ S = (B - S) \oplus S$$

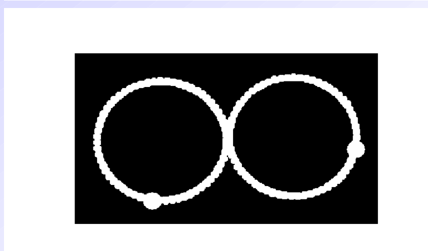


Gear Tooth Inspection

Sternberg 1985: use morphology to detect missing or broken watch gear teeth

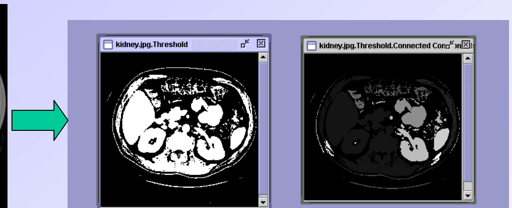


Original binary image

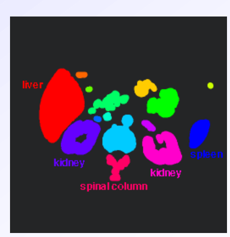


Detected defects

How did they do it?



Connected Components after thresholding



Connected Components after Morphology

Region Properties

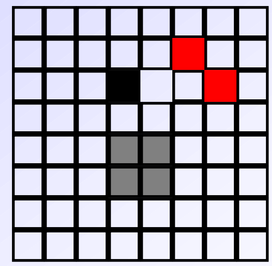
- Our goal is to recognize each connected component as one of a set of known objects
  - letters of the alphabet
  - good potatoes versus bad potatoes
- We need to associate measurements, or properties, with each connected component that we can compare against expected properties of different object types.

Properties

- Area
- Perimeter
- Compactness: P^2/A
  - smallest for a circle: $4\pi^2 r^2/\pi r^2 = 4\pi$
  - higher for elongated objects
- Properties of holes
  - number of holes
  - their sizes, compactness, etc.

How do we compute the perimeter of a connected component?

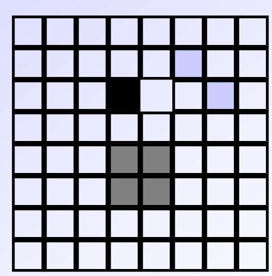
1. Count the number of pixels in the component adjacent to 0's
   - perimeter of black square would be 1
   - but perimeter of gray square, which has 4x the area, would be 4
   - but perimeter should go up as sqrt of area
2. Count the number of 0's adjacent to the component
   - works for the black and gray squares, but fails for the red dumbbell

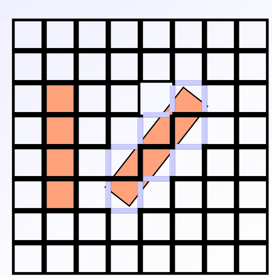


How do we compute the perimeter of a connected component?

3) Count the number of sides of pixels in the component adjacent to 0's
   - these are the **cracks** between the pixels
   - clockwise traversal of these cracks is called a crack code
   - perimeter of black is 4, gray is 8 and red is 8

- What effect does rotation have on the value of a perimeter of the digitization of a simple shape?
  - rotation can lead to large changes in the perimeter and the area!





A better (and universal)set of features

- An "ideal" set of features should be independent of
  - the position of the connected component
  - the orientation of the connected component
  - the size of the connected component
    - ignoring the fact that as we "zoom in" on a shape we tend to see more detail
- These problems are solved by features called moments

Central moments

- Let S be a connected component in a binary image
  - generally, S can be any subset of pixels, but for our application the subsets of interest are the connected components
- The (j,k)'th moment of S is defined to be

$$M_{jk}(S) = \sum_{(x,y)\in S} x^j y^k$$

Central moments

- M_{00} = the area of the connected component

$$M_{00}(S) = \sum_{(x,y)\in S} x^0 y^0 = \sum_{(x,y)\in S} 1 = |S|$$

- The center of gravity of S can be expressed as

$$\bar{x} = \frac{M_{10}(S)}{M_{00}(S)} = \frac{\sum x}{|S|}$$

$$\bar{y} = \frac{M_{01}(S)}{M_{00}(S)} = \frac{\sum y}{|S|}$$

Central moments

- Using the center of gravity, we can define the central (j,k)'th moment of S as

$$\mu_{jk} = \sum (x-\bar{x})^j (y-\bar{y})^k$$

- If the component S is translated, this means that we have added some numbers (a,b) to the coordinates of each pixel in S
  - for example, if a = 0 and b = -1, then we have shifted the component up one pixel

Central moments

- Central moments are not affected by translations of S. Let S'={(x', y'):x'=x+a, y'=y+b, (x,y) in S}
  - The center of gravity of S' is the c.o.g. of S shifted by (a,b)

$$\bar{x}(S') = \frac{\sum x'}{|S'|} = \frac{\sum (x+a)}{|S|} = \frac{\sum x}{|S|} + \frac{\sum a}{|S|} = \bar{x} + a$$

  - The central moments of S' are the same as those of S

$$\mu_{jk}(S') = \sum (x' - \bar{x}(S'))^j (y' - \bar{y}(S'))^k$$
$$= \sum (x + a - [\bar{x}(S) + a])^j (y + b - [\bar{y}(S) + b])^k$$
$$= \sum (x - \bar{x})^j (y - \bar{y})^k = \mu_{jk}(S)$$

Central moments

- The standard deviations of the x and y coordinates of S can also be obtained from central moments:

$$\sigma_x = \sqrt{\frac{\mu_{20}}{|S|}}$$

$$\sigma_y = \sqrt{\frac{\mu_{02}}{|S|}}$$

- We can then created a set of normalized coordinates of S that we can use to generate moments unchanged by translation and scale changes

$$\tilde{x} = \frac{x - \bar{x}}{\sigma_x} \qquad \tilde{y} = \frac{y - \bar{y}}{\sigma_y}$$

Normalized central moments

- The means of these new variables are 0, and their standard deviations are 1. If we define the normalized moments; η_{jk} as follows

$$\eta_{jk} = \frac{\sum \tilde{x}^j \tilde{y}^k}{M_{00}}$$

- then these moments are not changed by any scaling or translation of S
- Let S* = {(x*,y*): x* = ax + b, y* = ay + c, (x,y) in S}
  - if b and c are 0, then we have scaled S by a
  - if a is 0, then we have translated S by (b,c)

$$\eta_{jk}(S^*) = \frac{\sum (\frac{x^* - \overline{x(S^*)}}{\sigma_x(S^*)})^j (\frac{y^* - \overline{y(S^*)}}{\sigma_y(S^*)})^k}{|S|}$$

$$= \frac{\sum (\frac{a^j (x - \bar{x}(S))^j}{a^j \sigma_x{}^j(S)})(\frac{a^k (y - \bar{y}(S))^k}{a^k \sigma_y{}^k(S)})}{|S|}$$

$$= \eta_{jk}(S)$$

Invariant moments

- From normalized central moments we can obtain invariant moments

A set of seven invariant moments. Invariant to translation, rotation and scale changes. (Hu moments 1962)

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

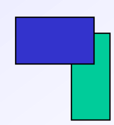
$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

Shortcomings of our machine vision system

- Object detection
  - thresholding will not extract intact objects in complex images
    - shading variations on object surfaces
    - texture
  - advanced segmentation methods
    - edge detection - locate boundaries between objects and background, between objects and objects
    - region analysis - find homogeneous regions; small combinations might correspond to objects.

Shortcomings of our machine vision system

- Occlusion
  - What if one object is partially hidden by another?
    - properties of the partially obscured, or occluded, object will not match the properties of the class model
  - Correlation - directly compare image of the “ideal” objects against real images
    - in correct overlap position, matching score will be high
  - Represent objects as collection of local features such as corners of a rectangular shape
    - locate the local features in the image
    - find combinations of local features that are configured consistently with objects



Shortcomings of our machine vision system

- Recognition of three dimensional objects
  - the shape of the image of a three dimensional object depends on the viewpoint from which it is seen
- Model a three dimensional object as a large collection of view-dependent models
- Model the three dimensional geometry of the object and mathematically relate it to its possible images
  - mathematical models of image geometry
  - mathematical models for recognizing three dimensional structures from two dimensional images

Shortcomings of our machine vision system

- Articulated objects
  - pliers
  - derricks
- Deformable objects
  - faces
  - jello
- Amorphous objects
  - fire
  - water

Sources

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  - L.S. Davis @UMD
  - G.C. Stockman @MSU