Digital Imaging and Multimedia
Point Operations in Digital Images

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Outlines

- Point Operations
- Brightness and contrast adjustment
- Auto contrast
- Histogram equalization
- Histogram specification

- Source: Burger & Burge "Digital Image Processing"

Point Operations

- Point Operations perform a mapping of the pixel values without changing the size, geometry, or local structure of the image
- Each new pixel value *I'(u,v)* depends on the previous value *I(u,v)* at the same position and on a mapping function *f*(.)
- The function *f*(.) is independent of the coordinates
- Such operation is called "homogeneous"

$$
\begin{aligned}
a' &\leftarrow f(a) \\
I'(u,v) &\leftarrow f\big(I(u,v)\big)
\end{aligned}
$$

Example of homogeneous point operations:

- Modifying image brightness or contrast
- Applying arbitrary intensity transformation (curves)
- Quantizing (posterizing) images
- Global thresholding
- Gamma correction
- Color transformations

- A nonhomogeneous point operation *g()* would also take into account the current image coordinate *(u,v)*

$$
\begin{aligned}
a' &\leftarrow g(a, u, v) \\
I'(u, v) &\leftarrow g\left(I(u, v), u, v\right)
\end{aligned}
$$

- Changing contrast and brightness

  $f_{\text{contr}}(a) = a \cdot 1.5 \qquad \text{and} \qquad f_{\text{bright}}(a) = a + 10$

- Limiting Results by Clamping

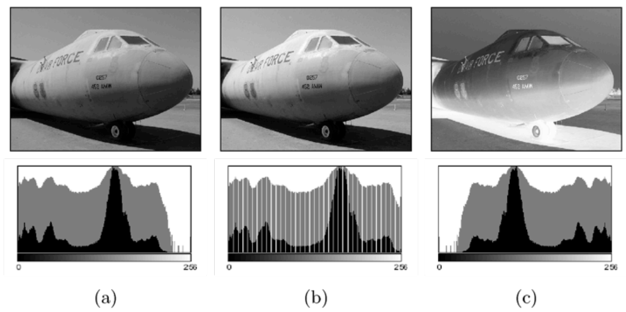
```
if (a > 255)  a = 255;
if (a < 0)  a = 0;
```

```
public void run(ImageProcessor ip) {
  int w = ip.getWidth();
  int h = ip.getHeight();

  for (int v = 0; v < h; v++) {
    for (int u = 0; u < w; u++) {
      int a = (int) (ip.get(u, v) * 1.5 + 0.5);
      if (a > 255)
        a = 255;    // clamp to maximum value
      ip.set(u, v, a);
    }
  }
}
```

- Inverting Images

$$f_{\text{invert}}(a) = -a + a_{\max} \;=\; a_{\max} - a$$



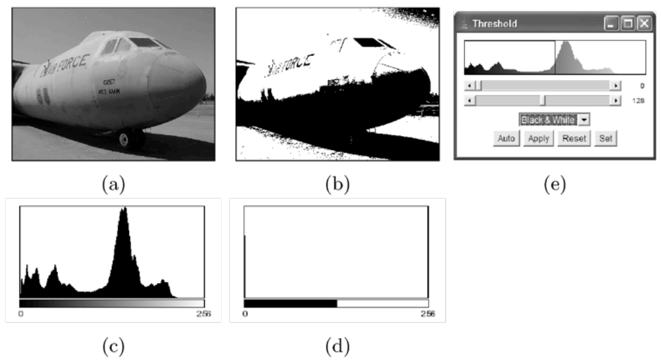
Threshold Operation

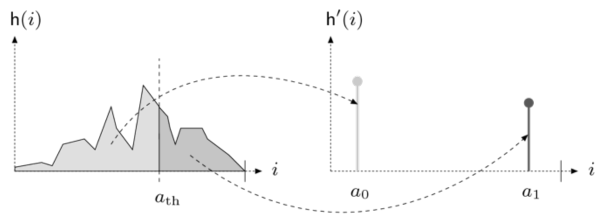
- Thresholding an image is a special type of quantization that separates the pixel values in two classes, depending on a given threshold value a_{th}
- The threshold function maps all the pixels to one of two fixed intensity values a_o, a_1

$$f_{\text{threshold}}(a) = \begin{cases} a_0 & \text{for } a < a_{\text{th}} \\ a_1 & \text{for } a \geq a_{\text{th}} \end{cases}$$

$$0 < a_{\text{th}} \leq a_{\max}$$

- Example: binarization: $a_o=0, a_1=1$

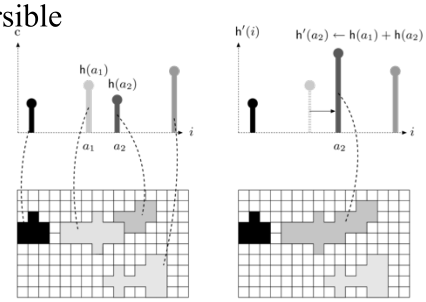




(a) (b)

Point Operations and Histograms

- The effect of some point operations on histograms are easy to predict: ex: increasing the brightness, raising the contrast, inverting an image
- Point operations can only shift and merge histogram entries
- Operations that result in merging histogram bins are irreversible



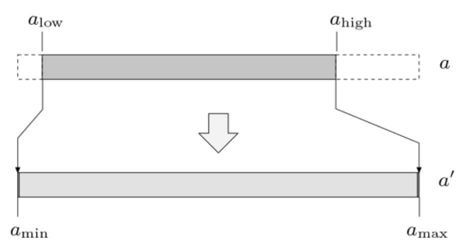
(a) (b)

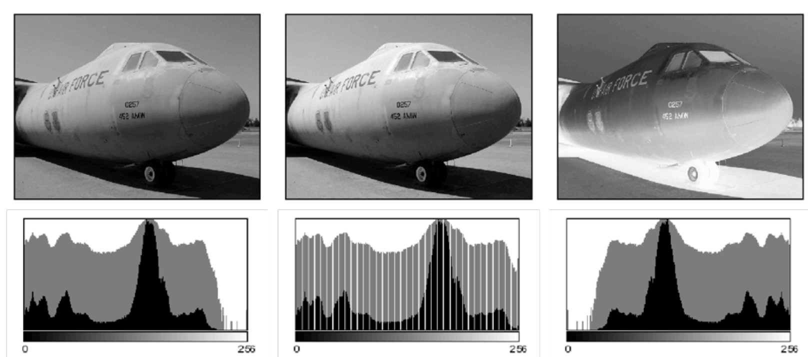
Automatic Contrast Adjustment

- Auto-contrast: a point operation that modifies the pixels such that the available range of values is fully covered.
- Linear stretching of the intensity range - can result in gaps in the new histogram

$$f_{\mathrm{ac}}(a) = a_{\mathrm{min}} + \left(a - a_{\mathrm{low}}\right) \cdot \frac{a_{\mathrm{max}} - a_{\mathrm{min}}}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

$$f_{\mathrm{ac}}(a) = (a - a_{\mathrm{low}}) \cdot \frac{255}{a_{\mathrm{high}} - a_{\mathrm{low}}}$$

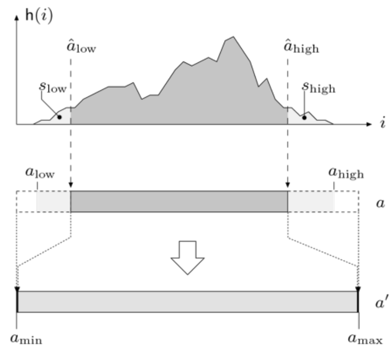




(a) (b) (c)

Better Auto-contrast

- It's better to map only a certain range of the values and get rid of the tails (usually noise) based on predefined percentiles (s_{low}, s_{high})



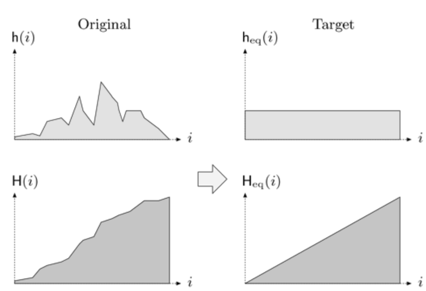
$$\hat{a}_{\text{low}} = \min\{\, i \mid \mathsf{H}(i) \geq M \cdot N \cdot s_{\text{low}} \,\}$$

$$\hat{a}_{\text{high}} = \max\{\, i \mid \mathsf{H}(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \,\}$$

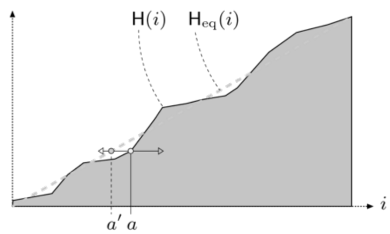
$$f_{\text{mac}}(a) = \begin{cases} a_{\min} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\min} + \left(a - \hat{a}_{\text{low}}\right) \cdot \dfrac{a_{\max} - a_{\min}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\max} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$

Histogram Equalization

- Adjust two different images in such a way that their resulting intensity distribution are similar
- Useful when comparing images to get rid of illumination variations
- The goal is to find and apply a point operation such that the histogram of the modified image approximates a uniform distribution.

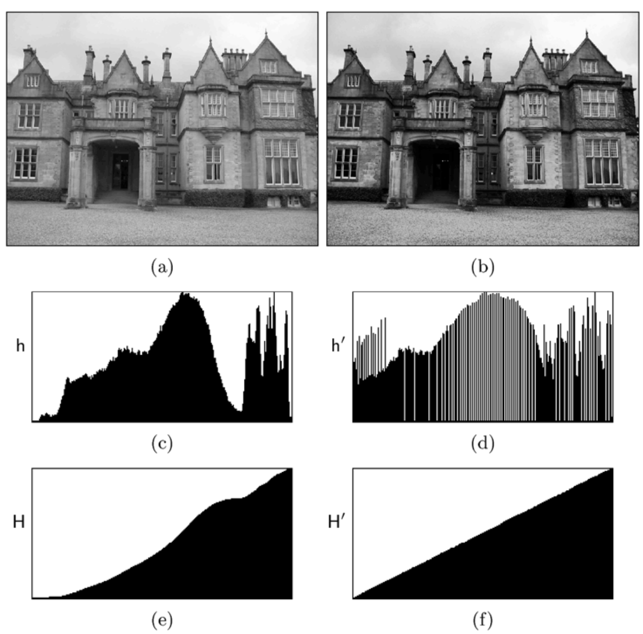


- Linear Histogram equalization



$$f_{\mathrm{eq}}(a) = \left\lfloor \mathsf{H}(a) \cdot \frac{K-1}{MN} \right\rfloor$$

Range is [0,K-1]



Histogram Specification

- Real images never show uniform distribution
- In most real images the distribution of pixel values is more similar to a Gaussian Distribution
- Histogram specification modifies the image to match an arbitrary intensity distribution, including the histogram of a given image.
- Also depends on the alignment of the cumulative histograms by applying a homogeneous point operation.

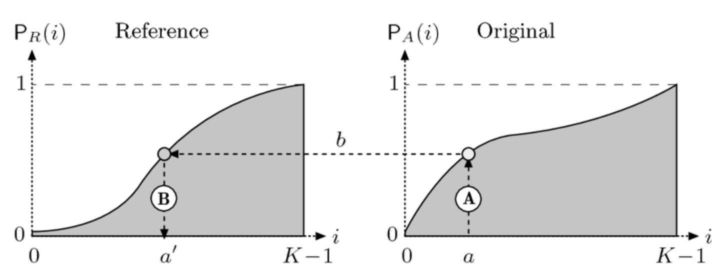
Histogram Specification

- Find a mapping such that

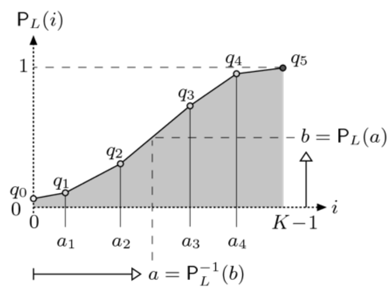
$$a' = f_{\mathrm{hs}}(a)$$

$$\mathsf{P}_{A'}(i) \approx \mathsf{P}_R(i) \qquad \text{for } 0 \le i < K$$

$$f_{\mathrm{hs}}(a) = a' = \mathsf{P}_R^{-1}\big(\mathsf{P}_A(a)\big)$$

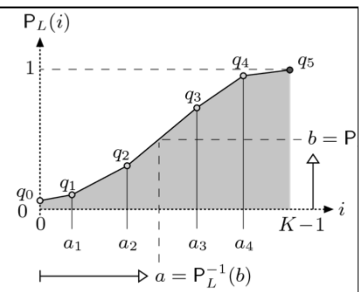


Adjusting piecewise linear distribution



$$\mathcal{L} = [\langle a_0, q_0 \rangle, \langle a_1, q_1 \rangle, \ldots \langle a_k, q_k \rangle, \ldots \langle a_N, q_N \rangle]$$

$$\langle 0, q_0 \rangle \qquad \text{and} \qquad \langle K-1, 1 \rangle$$



$$\mathsf{P}_L(i) = \begin{cases} q_m + (i - a_m) \cdot \dfrac{(q_{m+1} - q_m)}{(a_{m+1} - a_m)} & \text{for } 0 \le i < K-1 \\ 1 & \text{for } i = K-1 \end{cases}$$

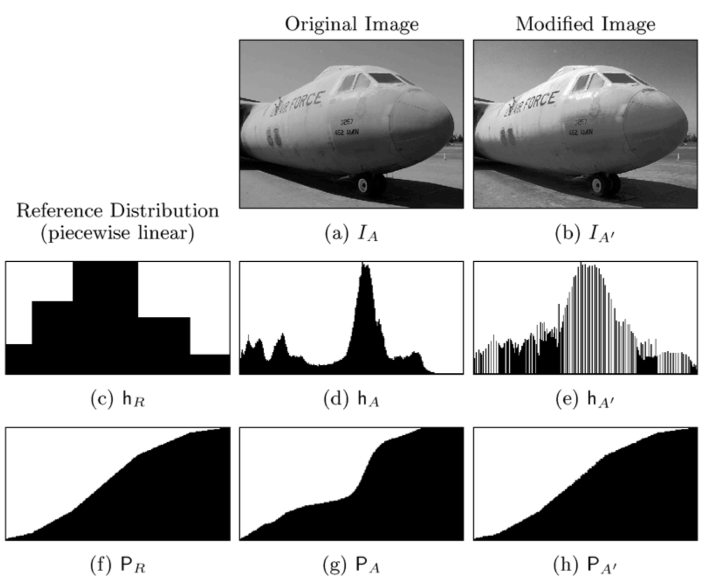
$$\mathsf{P}_L^{-1}(b) = \begin{cases} 0 & \text{for } 0 \le b < \mathsf{P}_L(0) \\ a_n + (b - q_n) \cdot \dfrac{(a_{n+1} - a_n)}{(q_{n+1} - q_n)} & \text{for } \mathsf{P}_L(0) \le b < 1 \\ K-1 & \text{for } b \ge 1 \end{cases}$$

$$n = \max\{ j \in \{0, \ldots N-1\} \mid q_j \le b \}$$

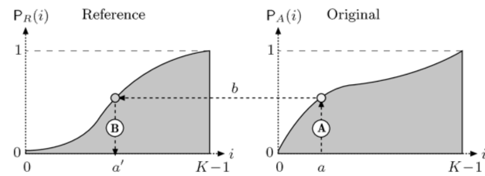
PIECEWISELINEARHISTOGRAM(h_A, $\mathcal{L}_R$)

h_A: histogram of the original image.
$\mathcal{L}_R$: reference distribution function, given as a sequence of $N+1$ control points $\mathcal{L}_R = [\langle a_0, q_0\rangle, \langle a_1, q_1\rangle, \ldots \langle a_N, q_N\rangle]$, with $0 \leq a_k < K$ and $0 \leq q_k \leq 1$.

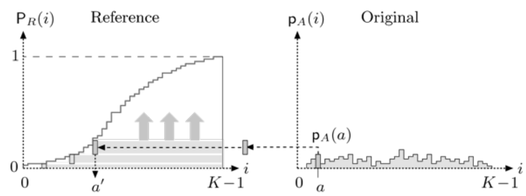
Let $K \leftarrow \text{Size}(\mathsf{h}_A)$
Let $\mathsf{P}_A \leftarrow \text{CDF}(\mathsf{h}_A)$ ▷ cdf for h_A (Alg. 5.1)
Create a table $f_{\text{hs}}[\,]$ of size K ▷ mapping function f_{hs}
for $a \leftarrow 0 \ldots (K-1)$ **do**
  $b \leftarrow \mathsf{P}_A(a)$
  if $(b \leq q_0)$ **then**
    $a' \leftarrow 0$
  else if $(b \geq 1)$ **then**
    $a' \leftarrow K-1$
  else
    $n \leftarrow N-1$
    while $(n \geq 0) \wedge (q_n > b)$ **do** ▷ find line segment in $\mathcal{L}_R$
      $n \leftarrow n-1$
    $a' \leftarrow a_n + (b-q_n) \cdot \frac{(a_{n+1} - a_n)}{(q_{n+1} - q_n)}$ ▷ see Eqn. (5.23)
  $f_{\text{hs}}[a] \leftarrow a'$
return f_{hs}.

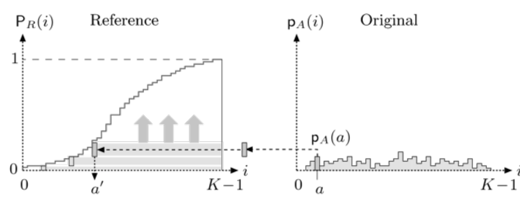


Adjusting to a given histogram



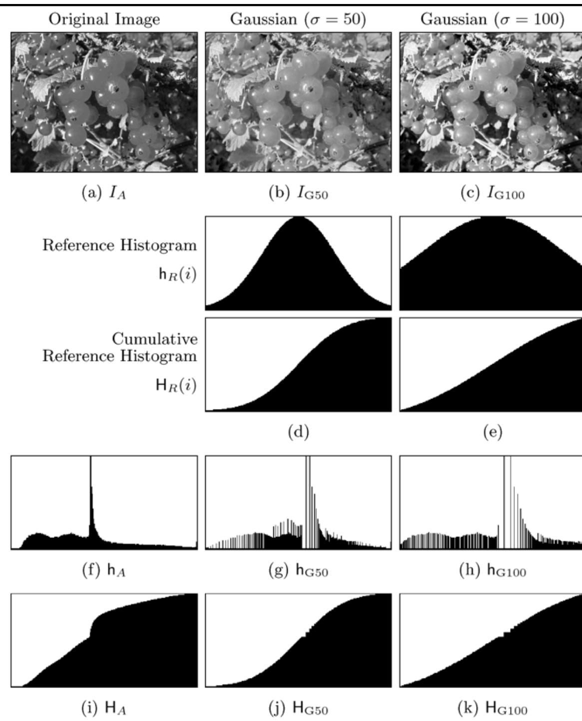
$$f_{\text{hs}}(a) = a' = \min\{\, j \mid (0 \le j < K) \wedge \big(\mathsf{P}_A(a) \le \mathsf{P}_R(j)\big) \}$$

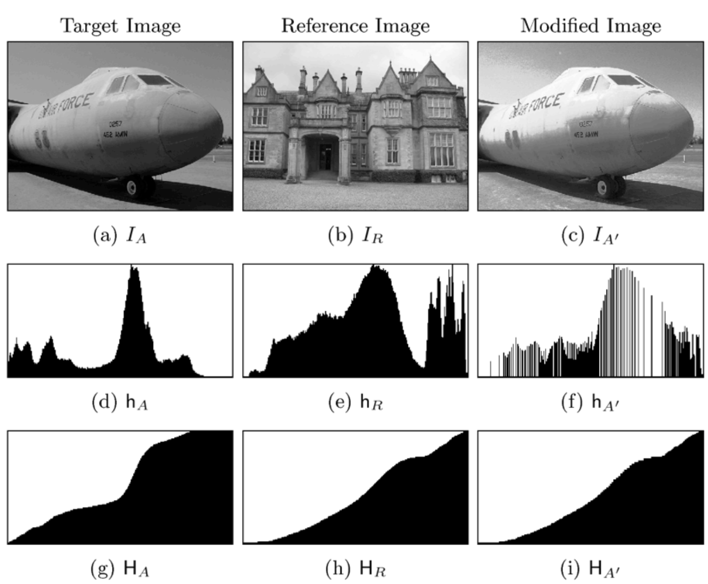





```
MatchHistograms(h_A, h_R)
    h_A: histogram of the target image
    h_R: reference histogram (of same size as h_A)

    Let K ← Size(h_A)
    Let P_A ← Cdf(h_A)                          ▷ cdf for h_A (Alg. 5.1)
    Let P_R ← Cdf(h_R)                          ▷ cdf for h_R (Alg. 5.1)
    Create a table f_hs[ ] of size K            ▷ pixel mapping function f_hs
    for a ← 0...(K−1) do
        j ← K−1
        repeat
            f_hs[a] ← j
            j ← j − 1
        while (j ≥ 0) ∧ (P_A(a) ≤ P_R(j))
    return f_hs.
```


Original Image
Gaussian ($\sigma = 50$)
Gaussian ($\sigma = 100$)
(a) I_A
(b) I_{G50}
(c) I_{G100}
Reference Histogram
$h_R(i)$
Cumulative
Reference Histogram
$H_R(i)$
(d)
(e)
(f) h_A
(g) h_{G50}
(h) h_{G100}
(i) H_A
(j) H_{G50}
(k) H_{G100}

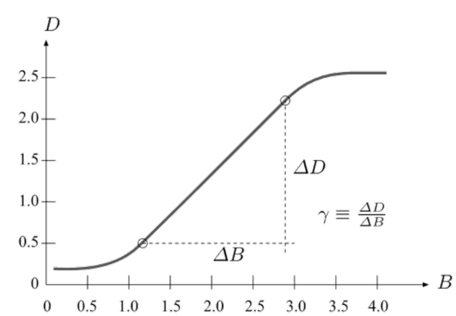

Target Image
Reference Image
Modified Image
(a) I_A
(b) I_R
(c) $I_{A'}$
(d) h_A
(e) h_R
(f) $h_{A'}$
(g) H_A
(h) H_R
(i) $H_{A'}$

Gamma Correction

- What is the relation between the amount of light falling onto a sensor and the "intensity" or "brightness" measured at the corresponding pixel.
- What is the relation between the intensity of a pixel and the actual light emanating from that pixel on the display, or toner particles in a printer?
- The relation between a pixel value and the corresponding physical quantity is usually complex and nonlinear.
- Approximation ?

What is Gamma?

- Originates from analog photography
- Exposure function: the relationship between the logarithmic light intensity and the resulting film density.
- Gamma is the slope of the linear range of the curve.
- The same in TV broadcasting

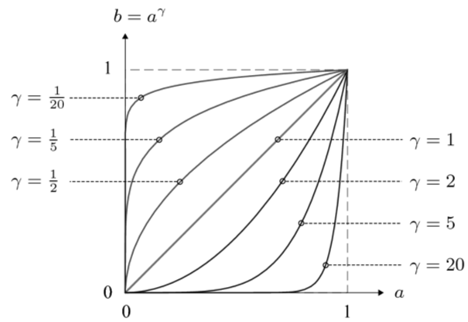


The Gamma function

- Gamma function is a good approximation for the exposure curve.
- The inverse of a Gamma function is another gamma function with

$$\bar{\gamma} = 1/\gamma$$

- Gamma of CRT and LCD monitors: 1.8-2.8 (typically 2.4)



$$b = f_\gamma(a) = a^\gamma \quad \text{for } a \in \mathbb{R}, \gamma > 0$$

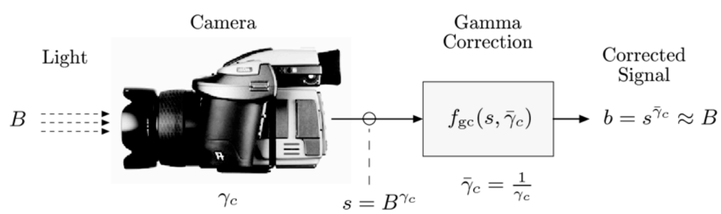
$$a = f_\gamma^{-1}(b) = b^{1/\gamma}$$

$$f_\gamma^{-1}(b) = f_{\bar{\gamma}}(b)$$

$$\bar{\gamma} = 1/\gamma$$

Gamma Correction

- Obtain a measurement b proportional to the original light intensity B by applying the inverse gamma function
- This is important to achieve a device independent representation



Gamma Correction