CS 206 , Spring 2007, Quiz 3 March 1^{st} , 2007

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In all the following problems we are dealing with an ordinary deck of cards (52 cards).

1) Two cards are randomly selected. What is the probability that they are both aces?

We have 4 choose 2 ways to select 2 aces out of 52 choose 2 ways, so the probability is $\frac{4.3}{52.51}$

2) Two cards are randomly selected. What is the probability that they form a blackjack ? (That is, two cards where one of the cards is an ace and the other one is a ten, a jack, a queen, or a king)

We have 4 ways to choose an ace and 16 ways to choose the other card, so the probability is $\frac{4.16}{C_4^{52}} = \frac{2.4.16}{52 \cdot 51}$

3) Two cards are randomly selected. What is the probability that they have the same value?

We have 13 different values to choose from and we need to choose 2 out of 4 in each group of values, so the probability is $\frac{13 \cdot C_2^4}{C_2^{52}} = \frac{13 \cdot 4 \cdot 3}{52 \cdot 51}$

4) If the deck of cards is dealt out, what is the probability that the fourteenth card dealt is an ace?

The fourteenth card is equally likely to be any of the 52 cards. So the probability is 4/52. another way to see this is to look at the 52! arrangements of cards, out of these there are 4.51! arrangements containing an ace as the 14 card. So the probability is $\frac{4 \cdot 51!}{52!} = \frac{4}{52}$

5) If the deck of cards is dealt out, what is the probability that the first ace occurs on the fourteenth card?

The first ace occurs on the fourteenth card means that we only have 48 cards to choose from in the first card followed by 47 for the second, …, then we have 4 ways to get an ace in the out of 39 cards for the 14^{th} card. so the probability is

$$\frac{48.47....36.4}{52.51....40.39}$$

6) If the deck of cards is shuffled, what is the probability that the top four cards have different denominations?

The top 4 cards can be chosen by C_4^{52} ways. We have C_4^{13} ways to choose the four denominations appearing on the top and for each denomination there are 4 ways to select one cards. So the probability is $\frac{C_4^{13}\cdot 4\cdot 4\cdot 4\cdot 4}{C_4^{52}} = \frac{13.12.11.10.4.4.4.4}{52.51.50.49}$

Another way to think about it is, we have 52 ways for the first card our of 52 then, we have 52-4=48 ways out of 51 ways to choose the second, 52-8=44 ways out of 50 to chose the third, 52-12=40 ways out of 49 ways to choose the forth to make that arrangement. The probability is $\frac{52.48.44.40}{52.51.50.49}$

7) If the deck of cards is shuffled, what is the probability that the top four cards have different suits?

This is very similar to part 6. The probability is $\frac{52.39.26.13}{52.51.50.49}$