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- 1) How many ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

Straightforward application of Theorem 2.

$r = 12, n = 21$, so we have $C(12+21-1, 12) = C(32, 12) = 225,792,840$ ways.

- 2) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 21$$

where x_1, x_2, x_3 , and x_4 are nonnegative integers?

Straightforward application of Theorem 2.

$r = 21, n = 4$, so we have $C(21+4-1, 21) = C(24, 21) = 2,024$ solutions.

- 3) How many solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 21$$

where x_1, x_2, x_3 , and x_4 are nonnegative integers?

This is equivalent to finding the solutions of the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$

and with an application of Theorem 2:

$r = 21, n = 5$, so we have $C(21+5-1, 21) = C(25, 21) = 12,650$ solutions.

- 4) How many solutions are there for the inequality in part (3) if $x_1 \geq 1$

In this case, the restriction removes one choice from 3).

So, now we have

$r = 21-1 = 20, n = 5$, and $C(20+5-1, 20) = C(24, 20) = 10,626$ solutions.

- 5) How many solutions are there for the inequality in part (3) if $x_1 \geq 2, x_2 \geq 2, x_3 \geq 2, x_4 \geq 2$

The restriction removes 8 choices from 3).

Thus, we have $r = 21 - 8 = 13, n = 5$, and $C(13+5-1, 13) = C(17, 13) = 680$ solutions.

- 6) How many solutions are there for the inequality in part (3) if $0 \leq x_1 \leq 10$

The restriction is the same as computing the total possible solutions without restriction minus the total solutions with the restriction $x_1 \geq 11$.

The total is $r = 21, n = 5, C(21+5-1, 21) = C(25, 21)$.

The total with restriction $x_1 \geq 11$ is: $r = 21 - 11 = 10, n = 5$, and $C(10+5-1, 10)$.

The solutions is thus $C(25, 21) - C(14, 10) = 11,649$ solutions.

- 7) How many ways are there to distribute 12 indistinguishable balls into six distinguishable boxes

This is the same as asking for the number of ways to choose 12 bins from the six given bins, with repetition allowed.

By Theorem 2, $r = 12, n = 6$, and $C(12+6-1, 12) = C(17, 12) = 6,188$ ways.