Valid / Invalid Arguments

week 5&6
10-1-1

Argument

$$\left.\begin{array}{l} H_1 ; \\ H_2 ; \\ H_3 ; \\ \vdots \\ H_n ; \end{array}\right\} \text{premises or Assumptions or Hypotheses}$$

$$\therefore C$$

therefore

Conclusion

Ex:

if it snows today then we will go skiing ;

it is snowing today ;

$\therefore$ we will go skiing

Argument form

$$p \rightarrow q;$$
$$p;$$
$$\therefore q$$

What is a valid argument?
invalid ?

An argument <u>form</u> is valid
means, no matter what statements
are substituted in its premises;
if all the premises are true
then the conclusion is true

An argument is valid means

It's form is valid

How to show that an argument form is valid ??

use truth table :

- Construct truth table for all the premises & the conclusion
- find critical raws (all premises are true)
- if in each critical raw the Conclusion is also true then the form is valid

o.w invalid

$p \vee (q \vee r)$

$\neg r$

$\therefore\ p \vee q$

premises ↓ ↓ Concl. ↓

p	q	r	$p \vee (q \vee r)$	$\neg r$	$p \vee q$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	T	F	T
T	F	F	T	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	F	T	F

Form is valid

$$
\begin{array}{l}
p_1 \\
p_2 \\
\vdots \\
p_n \\
\therefore\ q
\end{array}
$$

an argument form is valid iff

$$(p_1 \wedge p_2 \ \ldots\ldots \wedge p_n) \rightarrow q$$

is a tautology

Modus Ponens

"method of affirming"

$p \rightarrow q$

p

$\therefore q$

- if the last digit of ~~this~~ this number is $\emptyset$ then this number is divisible by 10
- the last digit of this number is 0

$\therefore$ this number is divisible by 10.

Modus Tollens

$$p \to q$$
$$\sim q$$
$$\therefore \sim p$$

- if x is divisible by 6 then it is divisible by 2
- x is not divisible by 2

$\therefore$ x is not divisible by 6

Disjunctive Addition

p

$\therefore p \vee q$

q

$\therefore p \vee q$

"generalization"

Jack is a junior

$\therefore$ Jack is an upperclassman

(jounior or senior)

Conjunctive Simplification

$$\begin{array}{l} p \wedge q \\ \therefore p \end{array} \qquad \begin{array}{l} p \wedge q \\ \therefore q \end{array}$$

\#

particularizing

Jack knows C++ and Java

∴ Jack knows Java

Disjunctive Syllogism

$p \vee q$

$\sim q$

$\therefore p$

$p \vee q$

$\sim p$

$\therefore q$

"rule one out"!

My key is in my Coat pocket or it is in my wallet

My key is not in my Coat pocket

$\therefore$ My key is in my wallet.

Hypothetical Syllogism

$p \to q$

$q \to r$

$\therefore\ p \to r$

Dilemma: Proof by Division into Cases

$p \vee q$

$p \to r$

$q \to r$

$\therefore\ r$

Fallacies

10-1-12

Converse Error

$$p \rightarrow q$$

$$q$$

$$\therefore\ p$$

wrong!!

Inverse Error

$$p \rightarrow q$$

$$\sim p$$

$$\therefore\ \sim q$$ wrong!!

Contradiction Rule

$$\sim P \rightarrow C \quad \text{(contradiction)}$$
$$\therefore\ P$$

$$P \rightarrow C$$
$$\therefore\ \sim P$$

Application: A More Complex Deduction

You are about to leave for school in the morning and discover you don't have your glasses. You know the following statements are true:

a. If my glasses are on the kitchen table, then I saw them at breakfast.

b. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.

c. If I was reading the newspaper in the living room, then my glasses are on the coffee table.

d. I did not see my glasses at breakfast.

e. If I was reading my book in bed, then my glasses are on the bed table.

f. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Where are the glasses?

Solution:

The glasses are on the coffee table. Here is a sequence of steps you might use to reach this answer, together with the rules of inference that allow you to draw the conclusion of each step.

1. The glasses are not on the kitchen table.	by (a), (d), and *modus tollens*
2. I did not read the newspaper in the kitchen.	by (f), (1), and *modus tollens*
3. I read the newspaper in the living room.	by (b), (2), and disjunctive syllogism
4. My glasses are on the coffee table.	by (c), (3), and *modus ponens*

Note that (e) was not needed to derive the conclusion. In mathematics as in real life, we frequently deduce a conclusion from just a part of the information available to us. ■

10-1-15

.3.11 Symbolizing a Situation to Find a Solution

Solve the problem of Example 1.3.10 symbolically.

Solution:

Let p = My glasses are on the kitchen table.
q = I saw my glasses at breakfast.
r = I was reading the newspaper in the living room.
s = I was reading the newspaper in the kitchen.
t = My glasses are on the coffee table.
u = I was reading my book in bed.
v = My glasses are on the bed table.

Then the statements of Example 1.3.10 translate into the following.

a. $p \to q$
b. $r \vee s$
c. $r \to t$
d. $\sim q$
e. $u \to v$
f. $s \to p$

The following deductions can be made.

1. $p \to q$ by (a)
   $\sim q$ by (d)
   $\therefore \sim p$ by *modus tollens*
2. $s \to p$ by (f)
   $\sim p$ by the conclusion of (1)
   $\therefore \sim s$ by *modus tollens*
3. $r \vee s$ by (b)
   $\sim s$ by the conclusion of (2)
   $\therefore r$ by disjunctive syllogism
4. $r \to t$ by (c)
   r by the conclusion of (3)
   $\therefore t$ by *modus ponens*

Hence t is true and the glasses are on the coffee table. ■