Section 1.6
Sets

A *set* is a collection or group of objects or *elements* or *members*. (Cantor 1895)

- A set is said to *contain* its elements.

- There must be an underlying universal set U, either specifically stated or understood.

Notation:

- list the elements between braces:

$$S = \{a, b, c, d\}=\{b, c, a, d, d\}$$

(Note: listing an object more than once does not change the set. Ordering means nothing.)

- specification by predicates:

$$S= \{x|\ P(x)\},$$

S contains all the elements from U which make the predicate *P* true.

- brace notation with ellipses:

$$S = \{\ . . . , -3, -2, -1\},$$

the negative integers.

Common Universal Sets

- R = reals
- N = natural numbers = {0,1, 2, 3, . . . }, the *counting* numbers
- Z = all integers = {. . , -3, -2, -1, 0, 1, 2, 3, 4, . .}
- Z^+ is the set of positive integers

Notation:

x is a member of S or x is an element of S:

$$x \in S.$$

x is not an element of S:

$$x \notin S.$$

SET THEORY

A set is a collection of <u>objects</u>
elements
members

$A = \{a, b, c\}$

All these are the same:

$\{a, b, c\}$ $\{b, c, a\}$ $\{b, b, a, c, a\}$

order is irrelevent

list elements between braces

Ex:

$V = \{a, e, i, o, u\}$

$O = \{1, 3, 5, 7, 9\}$

$X = \{a, 2,$ Fred, New Jersey$\}$

$I = \{0, 1, 2, 3, \ldots\}$

$$a \in S$$

a is an element of S

a is a member of S

a is in S

a belongs to S

$$a \notin S$$

$$1 \in \{3, 1, 2\}$$

$$u \in \{v, u, x, w\}$$

$$r \notin \{a, b, c\}$$

element

Set

principle of specification (intension)

$$A = \{ x \mid P(x) \}$$

the set of All — such that

$$O = \{1, 3, 5, 7, 9\}$$

$O = \{ x \mid x$ is an odd ~~number~~ positive integer less than 10 $\}$

$\mathbb{R} = \{ x \mid x$ is a real number $\}$

$$A = \{ x \in S \mid P(x) \}$$

$\mathbb{R}$: real numbers

$\mathbb{Z}$: set of all integers (Zahlen)

$\mathbb{Q}$: rational numbers

$\mathbb{Z}^{+}$ $\mathbb{Z}^{-}$ $\mathbb{Z}^{\text{non neg.}}$ . . .

$\{x \in \mathbb{R} \mid -2 < x < 5\}$

$\{x \in \mathbb{Z} \mid -2 < x < 5\}$

$= \{-1, 0, 1, 2, 3, 4\}$

$\{x \in \mathbb{Z}^+ \mid -2 < x < 5\}$

$= \{1, 2, 3, 4\}$

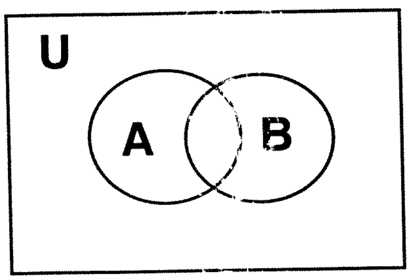
$\{1, 3, 5, 7, 9, 11, \ldots\}$

ellipses

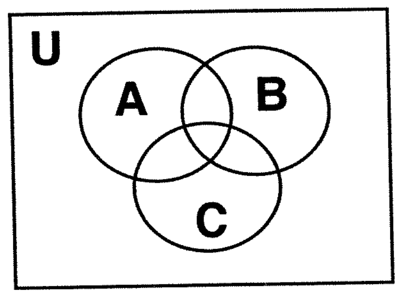
Venn Diagrams

A useful geometric visualization tool (for 3 or less sets)

- The Universe U is the rectangular box
- Each set is represented by a circle and its interior
- All possible combinations of the sets must be represented



For 2 sets



For 3 sets

Shade the appropriate region to represent the given set operation.

Two Sets are equal if and only if they have the same elements

$A = \{a, b, c\}$

$B = \{b, c, a\}$

$A = B$

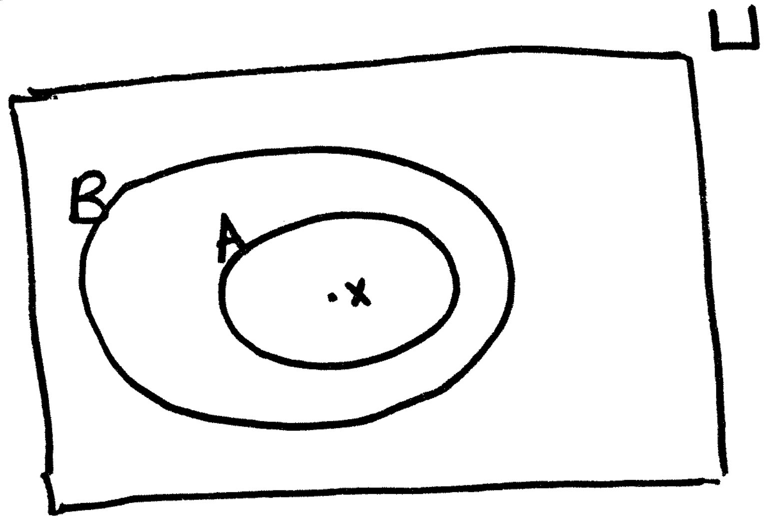
Any element in A is also an element in B

<u>And</u>

Any element in B is also an element in A

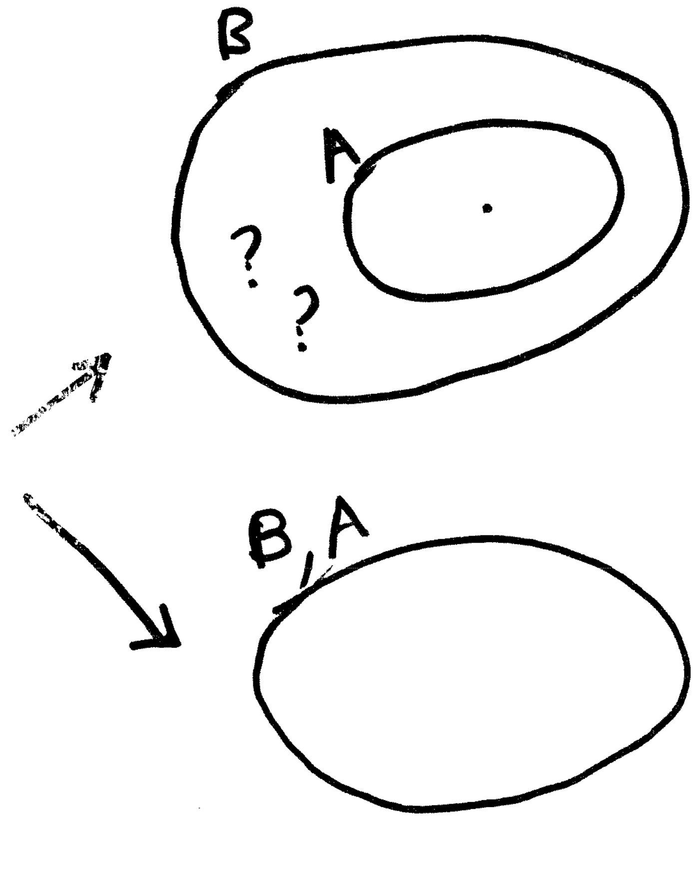
subsets

$A \subseteq B$



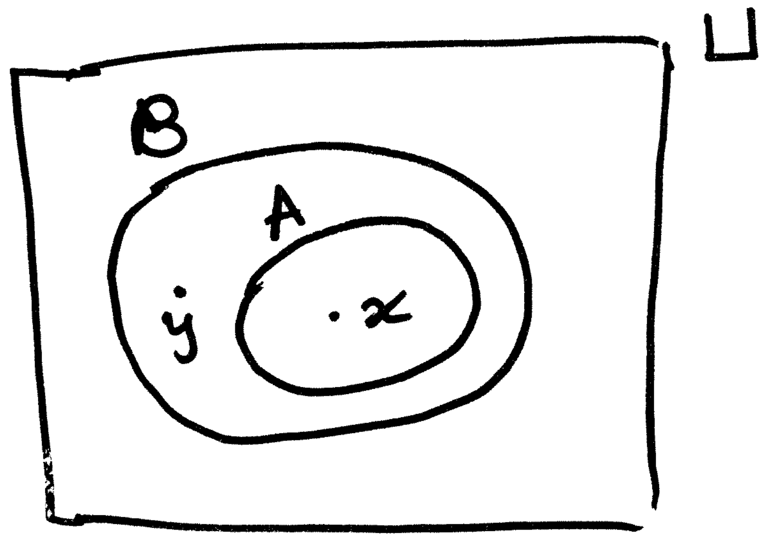
A is a subset of B : every element of A is also an element of B.

$A \subseteq B$



Proper Subset

A is a proper subset of B



every element in A is also an element in B

<u>and</u>

there is at least one element in B that is not in A.

<u>Note</u>

Every set is a subset of itself

$A \subseteq A$

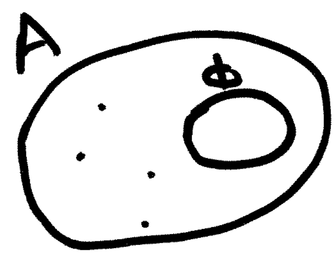
Empty Set ϕ

A set that doesn't contain any members

$\phi = \{ \}$

Also called : null set - void set

ϕ is a subset of every set



why?

All the elements in ϕ are also elements in any other set !!

True or False

$x \in \{x\}$	T
$\{x\} \subseteq \{x\}$	~~F~~ T
$\{x\} \in \{x\}$	F
$\{x\} \in \{\{x\}\}$	T
$\phi \subseteq \{x\}$	T
$\phi \in \{x\}$	F

Power Set

The set of all subsets of a set A is called the power set of A. denoted $P(A)$

Ex: $A = \{a, b\}$

what are the possible subsets of A

$\{a\}$

$\{b\}$

$\{a, b\}$

ϕ

$$P(A) = \{ \{a\}, \{b\}, \{a,b\}, \phi \}$$

Note:

Power set is a set of sets

Definition: The number of (distinct) elements in A, denoted |A|, is called the *cardinality* of A.

If the cardinality is a natural number (in N), then the set is called *finite*, else *infinite*.

Example:

A = {a, b},

$|\{a, b\}| = 2,$

$|P(\{a, b\})| = 4.$

A is finite and so is P(A).

Useful Fact: $|A|=n$ implies $|P(A)| = 2^n$

N is infinite since |N| is not a natural number. It is called a *transfinite cardinal number.*

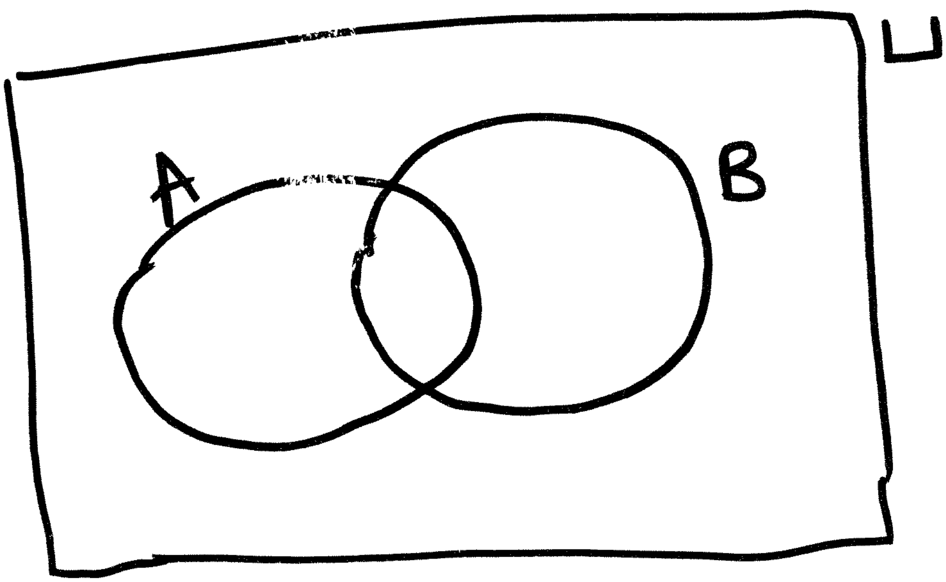
Note: Sets can be both members and subsets of other sets.

Set Union

The union of the sets A and B,
denoted by $A \cup B$,
is the set that contains those
elements that are either in A or in B
or in both.

Ex : $A = \{a, b, c\}$
$B = \{b, c, d, e\}$

$A \cup B = \{a, b, c, d, e\}$



mark $A \cup B$

Set Intersection

The intersection of the sets A & B,

denoted by $A \cap B$,

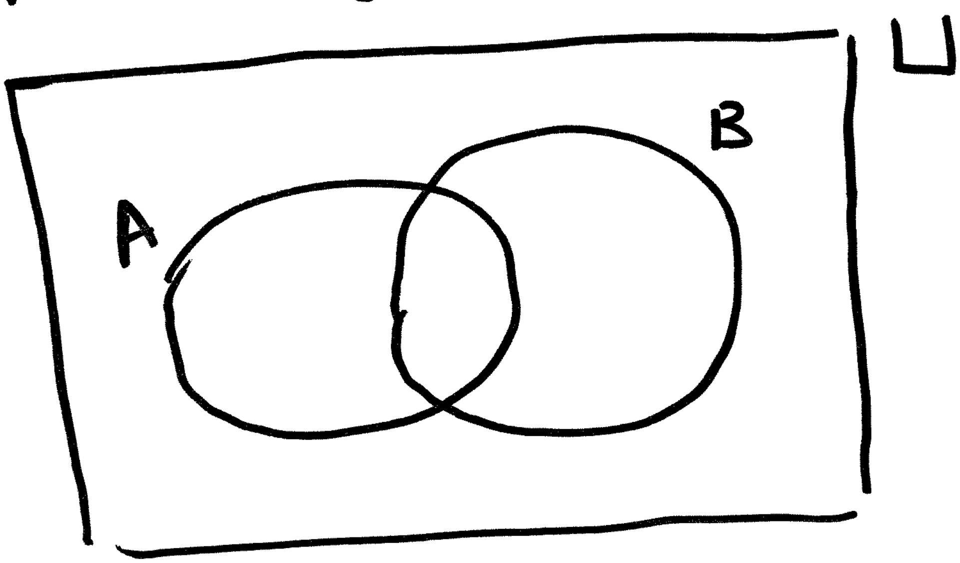
is the set containing those elements in both A and B (Common elements)

Ex:

$A = \{a, b, c\}$

$B = \{b, c, d, e\}$

$A \cap B = \{b, c\}$



mark $A \cap B$

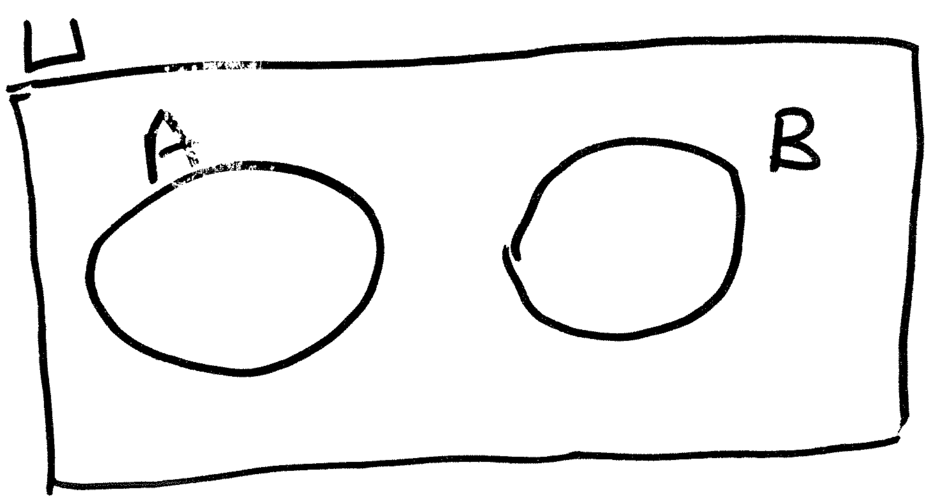
Disjoint Sets

Two sets are called disjoint if their intersection is the empty set (have no common elements)

Ex: $A = \{a, b, c\}$
$B = \{d, e, f\}$

$A \cap B = \{\ \} = \phi$

A and B are disjoint sets



Set Difference

the difference of A and B,

denoted by $A - B$,

is the set containing elements that are in A but not in B

Similarly $B - A$: set containing elements in A but not in B

Ex: $A = \{a, b, c\}$

$B = \{b, c, d, e\}$

$A - B = \{a\}$

$B - A = \{d, e\}$

$A - B$ is also called complement of B w.r.t. A

Complement

The complement of A, denoted by $\bar{A}$, is the complement of A w.r.t. U

i.e. $\bar{A} = U - A$

(the elements that are not in A)

Ex: $A = \{a, e, i, o, u\}$

what is $\bar{A}$ (where the Universal set is the English alphabet)

$\bar{A} = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v,$
$w, x, y, z\}$

Cartesian Product

The Cartesian Product of A and B,
denoted by $A \times B$,
is the set of all ordered pairs (a,b)
where $a \in A$, $b \in B$

Ex : $A = \{a, b\}$
$B = \{b, c, d\}$

$A \times B = \{(a, b), (a, c), (a, d),$
$(b, b), (b, c), (b, d)\}$

Examples: U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A= {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}. Then

- $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8}
- $A \cap B$ = {4, 5}
- $\overline{A}$ = {0, 6, 7, 8, 9, 10}
- $\overline{B}$ = {0, 1, 2, 3, 9, 10}
- $A - B$ = {1, 2, 3}
- $B - A$ = {6, 7, 8}
- $A \oplus B$ = {1, 2, 3, 6, 7, 8}

TABLE 1 **Set Identities.**

Identity	*Name*
$A \cup \emptyset = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws

What can you say about the sets A, B if the following is true

$A \cup B = A$

$B \subseteq A$

$A \cap B = A$

$A \subseteq B$

$A - B = A$

$A \cap B = \Phi$ disjoint

$A \cap B = B \cap A$

—

$A - B = B - A$

$\downarrow \quad \downarrow$

$\Phi \quad \Phi$

$A = B$