TABLE 1 **Quantifiers.**

Statement	*When True?*	*When False?*
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

TABLE 2 **Quantifications of Two Variables.**

Statement	*When True?*	*When False?*
$\forall x \forall y\, P(x, y)$ $\forall y \forall x\, P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y\, P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y\, P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y\, P(x, y)$ $\exists y \exists x\, P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

TABLE 3 **Negating Quantifiers.**

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg \exists x\, P(x)$	$\forall x \neg P(x)$	$P(x)$ is false for every x.	There is an x for which $P(x)$ is true.
$\neg \forall x\, P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

$C(x)$ is "x has a Computer"

$F(x,y)$: "x and y are friends"

U : set of all students in your school

$$\forall x\,(C(x) \lor \exists y\,(C(y) \land F(x,y)))$$

for every student x in your school (

x has a Computer

or

there is a student y (

such that

y has a computer

and

x and y are friend)

)

---

every students in your school has a Computer or has a friend who has a Computer

U: all students in your school

F(a, b): a and b are friends

$$\exists x \forall y \forall z (((F(x,y) \land F(x,z) \land (y \neq z)) \to \neg F(y,z))$$

there is a student x such that
for all students y and all students z
if (x and y are friends and
and z are friends and
y is not the same as z)

then
y and z are not friends

There is a student none of whose friends are also
friends with each other

English To logic

1- "Some students in this class has visited Mexico"

2- "Every student in this class has visited either Canada or Mexico"

U : set of students in this class

$M(x)$: x has visited Mexico

$C(x)$: x has visited Canada

1- $\exists x \; M(x)$

2- $\forall x \; (C(x) \vee M(x))$

Everyone has exactly one best friend.

let $B(x,y)$ be " y is the best friend of x "

for every person x there is another person y

such that y is the best friend of x

and

that if z is a person other than y

then z is not the best friend of x

$$\forall x\ \exists y\ \forall z\ (B(x,y) \land ((z \neq y) \rightarrow \neg B(x,z)))$$

"If somebody is female and is a parent then this person is someone's mother"

let

$F(x)$: "x is female"

$P(x)$: "x is a parent"

$M(x,y)$: "x is the Mother of y"

U : All people

$$\forall x\ ((F(x) \wedge P(x)) \longrightarrow \exists y\ M(x,y))$$

All hummingbirds are richly Colored

No large birds live on honey

Birds that do not live on honey are dull in Color

Hummingbirds are small

Let

U: all birds

$P(x)$: x is a hummingbird

$Q(x)$: x is Large

$R(x)$: x lives on honey

$S(x)$: x is richly Colored

$$\forall x \ (P(x) \rightarrow S(x))$$

$$\neg \exists x \ (Q(x) \wedge R(x))$$

$$\forall x \ (\neg R(x) \longrightarrow \neg S(x))$$

$$\forall x \ (P(x) \longrightarrow \neg Q(x)$$

"All lions are fierce"

"Some lions do not drink coffee"

"Some fierce creatures do not drink coffee"

Let

U: set of all creatures

$P(x)$: x is a lion

$Q(x)$: x is fierce

$R(x)$: x drinks coffee

$$\forall x \; (P(x) \longrightarrow Q(x))$$

$$\exists x \; (P(x) \wedge \neg R(x))$$

$$\exists x \; (Q(x) \wedge \neg R(x))$$

Lewis Carroll (C.L. Dodgson) 1832 – 1898

Symbolic logic

Let $L(x,y)$ be "x loves y"

U : all people

- Everybody loves Jerry

$$\forall x \ L(x, \text{Jerry})$$

- Everybody Loves Somebody

$$\forall x \ \exists y \ L(x,y)$$

- There is somebody whom everybody lover

$$\exists x \ \forall y \ L(y,x)$$

$$\exists y \ \forall x \ L(x,y)$$

- Nobody loves everybody

There does not exist any body who loves Everybody

$$\neg \exists x \ \forall y \ L(x,y)$$

for each person we can find someone whom he/she does not love

$$\forall x \ \exists y \ \neg L(x,y)$$

- There is somebody whom Jerry does not love

$$\exists x \quad \neg L(\text{Jerry}, x)$$

- There is somebody whom no one loves

$$\exists x \, \forall y \; \neg L(y, x)$$

- There is exactly one person whom everybody loves

$$\exists! \, x \, \forall y \, L(y, x)$$

Can you writ it without $\underline{\exists!}$??

- There are exactly two people whom Jerry loves

$$\exists x \, \exists y \, (x \neq y \wedge L(\text{Jerry}, x) \wedge L(\text{Jerry}, y)$$
$$\wedge \forall z \, (L(\text{Jerry}, z) \rightarrow z = x \vee z = y))$$

- Everyone loves himself or herself

$$\forall x \quad L(x,x)$$

- There is someone who loves no one besides himself or herself

$$\exists x \quad L(x,x) \wedge (\forall y \; (L(x,y) \rightarrow y = x))$$

$$\exists x \; \forall y \; (L(x,y) \leftrightarrow x = y)$$