

## Propositions

A **proposition** is a statement that is either true, T, or false, F.

1. Washington D.C. is the capitol of the US
2. Toronto is the capitol of Canada
3.  $1 + 1 = 2$
4.  $2 + 2 = 3$

Propositions 1 and 3 are True, whereas 2 and 4 are False. This is called the **truth value**.

The following are not propositions:

1. What time is it ?
2. Read this carefully.
3.  $x + 1 = 2$
4.  $x + y = z$

what about: "Today is Thursday "

## Compound propositions

Producing new propositions from those we have. We use **Truth Tables** to Let  $p$  and  $q$  be propositions.

**negation** Let  $p$  be a proposition. The negation of  $p$  is denoted by  $\neg p$  "not  $p$ " or "It is not the case that  $p$ ". If  $p$  is true then  $\neg p$  is false. If  $p$  is false then  $\neg p$  is true.

**conjunction** The conjunction of  $p$  and  $q$  is the proposition " **$p$  and  $q$** " denoted  $p \wedge q$ . This proposition is true when both  $p$  and  $q$  are true and false otherwise.

**disjunction** The disjunction of  $p$  and  $q$  is the the proposition " **$p$  or  $q$** " denoted  $p \vee q$ . This proposition is false when both  $p$  and  $q$  are false and true otherwise.

**exclusive or** The exclusive or of  $p$  and  $q$ , denoted by  $p \oplus q$ , is the proposition that is true when exactly one of  $p$  and  $q$  is true and is false otherwise. ( $p$  and  $q$  disagree in their truth value)

**implication**  $p \longrightarrow q$  ( $p$  implies  $q$ ) is the proposition that is false when  $p$  is true and  $q$  is false and true otherwise.

- if  $p$  then  $q$
- $q$  if  $p$
- $p$  is sufficient for  $q$
- $q$  whenever  $p$
- $q$  is necessary for  $p$
- $p$  only if  $q$

$p$  is called the hypothesis and  $q$  is called the conclusion

”If it is sunny today, then we will go to the beach”

The **converse** of  $p \longrightarrow q$  is the proposition  $q \longrightarrow p$ .

”If we go to the beach then it is sunny today”

The **contrapositive** of  $p \longrightarrow q$  is the proposition  $\neg q \longrightarrow \neg p$ .

”If we do not go to the beach then it is not sunny today”