CS538 Spring 98
Z GU
Feb. 9

Translational Methods:
Theorem: If $E \neq NE$ then $P \neq NP$
Proof:
Assume $P = NP$, let $A \in NE$. Here is an E algorithm for A.
On input X, accept $X \iff 2^X \in A'$
$X \in \sum^*$ represents a number in N.
$2^X = \underbrace{1000\ldots0}_{X\ zeros}$
Let $A' = \{2^X : X \in A\}$
Claim: $A' \in NP$
Construct a Turing Machine M' that accepts 2^X in polynomial time:
On input $\underbrace{100\ldots0}_{X\ zeros}$, run NE algorithm on X.
Since $P = NP$, then there exists a deterministic Turing Machine D' that accepts A' in polynomial time p'. We construct a deterministic Turing Machine D that accepts A in exponential time:
On input X, construct $\underbrace{100\ldots0}_{X\ zeros}$, run D' on this input.
The time complexity of D is $2^{O(n)}$ for constructing $\underbrace{100\ldots0}_{X\ zeros}$ $+\ p'(2^{O(n)})$. $\square$

Definition: The f-padded version of A is:
$$A' = \{X\#^{f(|X|)} : X \in A\}$$

Theorem:
Let t_1, t_2, f be time-constructible,
If $DTIME(t_1(n)) = DTIME(t_2(n))$
then $DTIME(t_1(f(n))) = DTIME(t_2(f(n)))$

Proof: Assume $t_2 < t_1$, then $DTIME(t_2(f(n))) \subseteq DTIME(t_1(f(n)))$
Let $A \in DTIME(t_1(f(n)))$, $A' = \{X\#^{f(|X|)-|X|} : X \in A\}$,
we claim: $A' \in DTIME(t_1(n))$
Obviously, $|X\#^{f(|X|)-|X|}| = f(|X|)$
We construct a deterministic Turing Machine D' that decides A' in time $DTIME(t_1(n))$.

On input $X\#^{f(|X|)-|X|}$

check if the input is in correct format,

run our $DTIME(t_1(f(n)))$ algorithm on the prefix.

Since $DTIME(t_1(n)) = DTIME(t_2(n))$, there exists a deterministic Turing Machine that decides A' in time $t_2(n)$.

We need to show $A \in DTIME(t_2(f(n)))$.

Construct a deterministic Turing Machine that decides A in time $t_2(f(n))$.

On input X

build $X\#^{f(|X|)-|X|}$

run $DTIME(t_2(n))$ algorithm on that string.

Therefore $DTIME(t_1(f(n))) \subseteq DTIME(t_2(f(n)))$. $\square$

Downward Seperation:

One example:

Claim: $DTIME(n^2) \subset DTIME(n^2(\log n)^{2/3})$

Assume they are equal, let $f(n) = 2^{n/2}$, we get

$DTIME(2^n n^{2/3}) = DTIME(2^n)$,

Apply $f(n) = 2^n + 2n/3$ again, we get

$DTIME(2^{2^n} 2^{2n/3}(f(n))^{2/3}) = DTIME(2^{2^n})$

but by the time hierarchy theorem,

$DTIME(2^{2^n} 2^{2n/3}(f(n))^{2/3}) \supset DTIME(2^{2^n})$

Contradiction. $\square$