Recitation 4

Computer Architecture (section 1)

Representing Signed Integers

11000000111001

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$2^{13}+2^{12}+2^{5}+2^{4}+2^{3}+2^{0}$

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Sign magnitude encoding

Extra bit to denote sign

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111000000111001 $\longrightarrow$ -12345

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Range:

$[-2^{n-1}-1, 2^{n-1}-1]$

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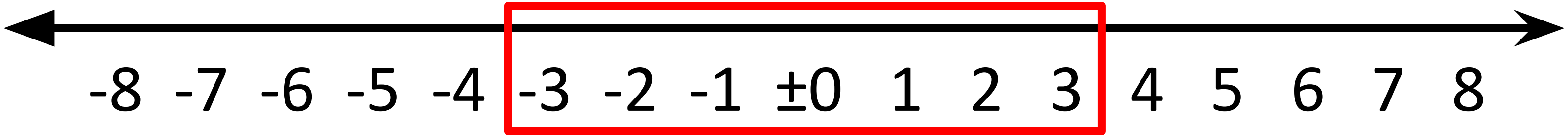
With 3 bits: *sign magnitude encoding*



Representing Signed Integers

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With 3 bits: *sign magnitude encoding*

-8 -7 -6 -5 -4 -3 -2 -1 ±0 1 2 3 4 5 6 7 8

Now

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Representing Signed Integers

Two's complement encoding

$$0110000000111001 \rightarrow 12345$$

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$101.011 \rightarrow 2^2 + 2^0$

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$101.011 \longrightarrow 2^2+2^0+1/2^2+1/2^3 = 4 + 1 + 1/4 + 1/8 = 5 + 3/8$

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Repeating pattern

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2 x 0.4 = 0.8 **LSB**

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LSB

0.01(1100)

Repeating pattern

Fixed Point Representation

Radix always at same index

1101.1001

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Problem: What if we need more or less precision?

1101.0000

0000.0101

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Problem: What if we need more or less precision?

1101.0000 — Wasted

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Form: $(-1)^S \times M \times 2^E$

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Spec: 9 bit FP with 4 bit exponent and 4 bit mantissa. Convert 23.8 to FP

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$10111.(1100) = 1.0111(1100) \times 2^4$

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$11 \longrightarrow 1011$

0 1011 011111001

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Round to nearest, ties to even

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- Special values
  - Exponent field all 1’s
  - Mantissa: All 0 $\longrightarrow$ $\pm\infty$, non-zero $\longrightarrow$ NaN