Recitation 3

Computer Architecture (section 1)

Number Systems

- A standard way to represent values.
  - You are all familiar with the *decimal number system.*
  - More importantly, we will discuss *positional number systems.*
- *Base* of a number system refers to the number of symbols.

$2{,}839{,}239{,}794_{10}$

110100010111_{2}

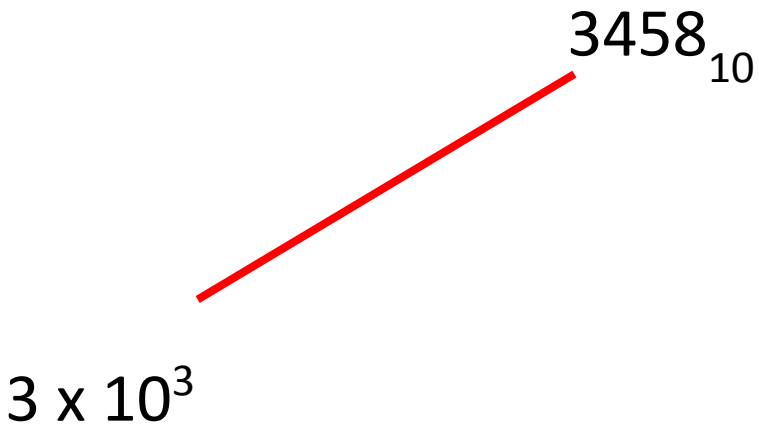
$0123456789ABCDEF_{16}$

1111111_{1}

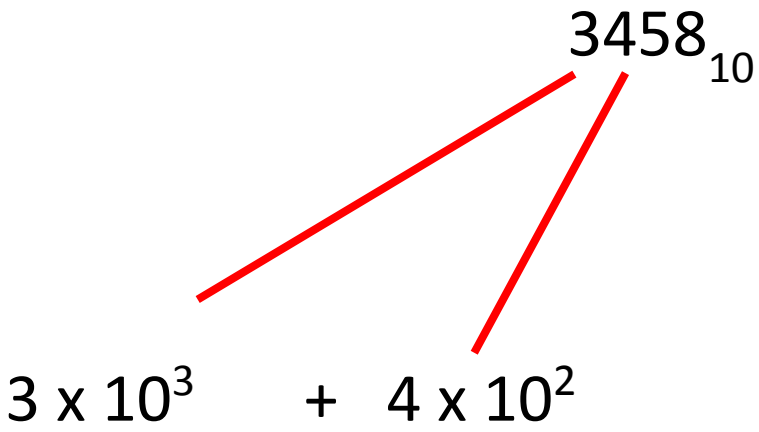
The Decimal Number System

3458_{10}

The Decimal Number System



The Decimal Number System



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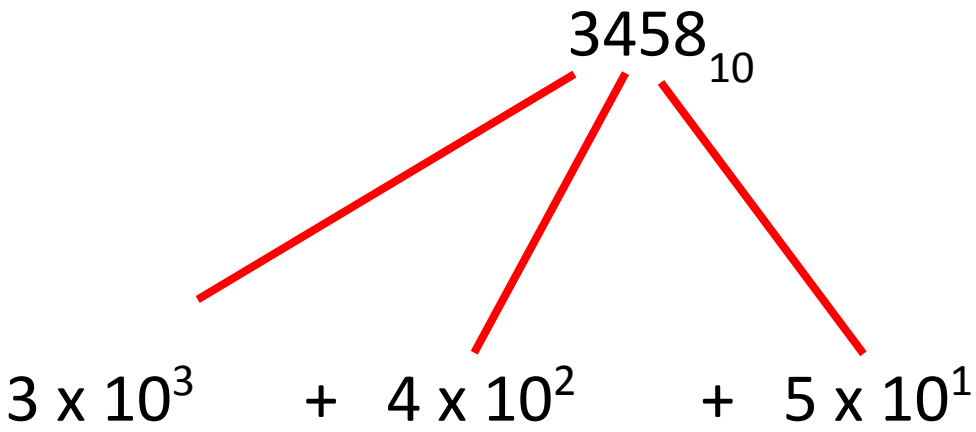
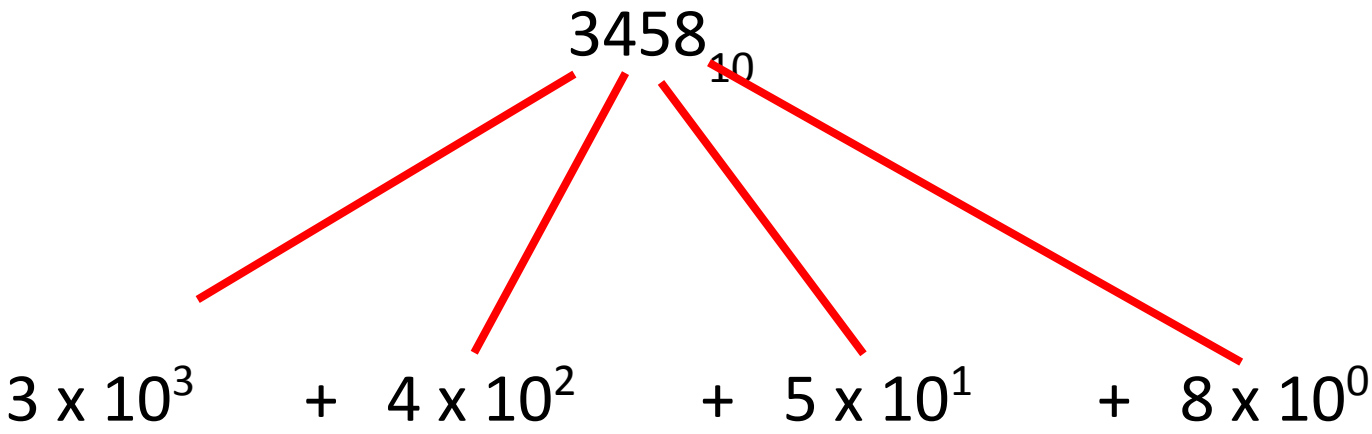


The Decimal Number System



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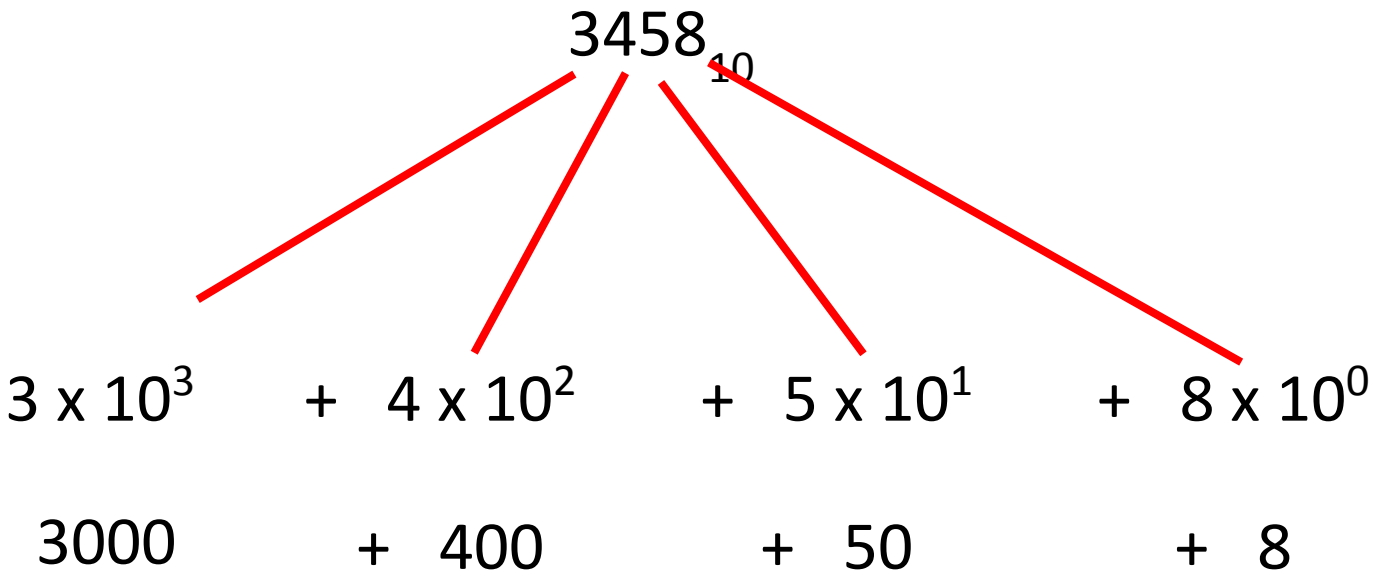


Decimal to Binary Example

91_{10}

Decimal to Binary Example

Just keep dividing by 2

91_{10}

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

11 **R** 0

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

11 **R** 0

5 **R** 1

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

11 **R** 0

5 **R** 1

2 **R** 1

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

11 **R** 0

5 **R** 1

2 **R** 1

1 **R** 0

Decimal to Binary Example

Just keep dividing by 2

91_{10}

45 **R** 1

22 **R** 1

11 **R** 0

5 **R** 1

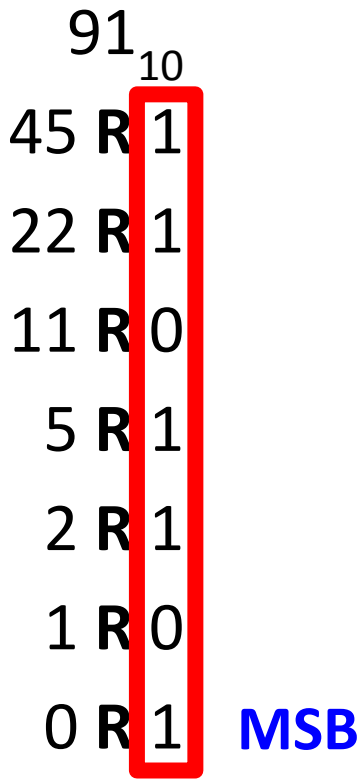
2 **R** 1

1 **R** 0

0 **R** 1

Decimal to Binary Example

Just keep dividing by 2



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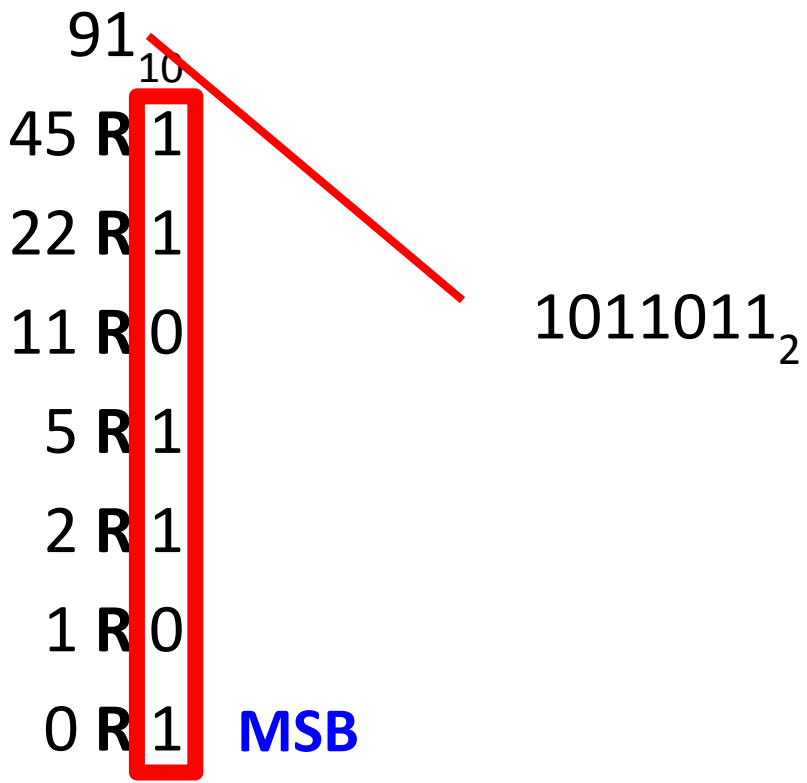
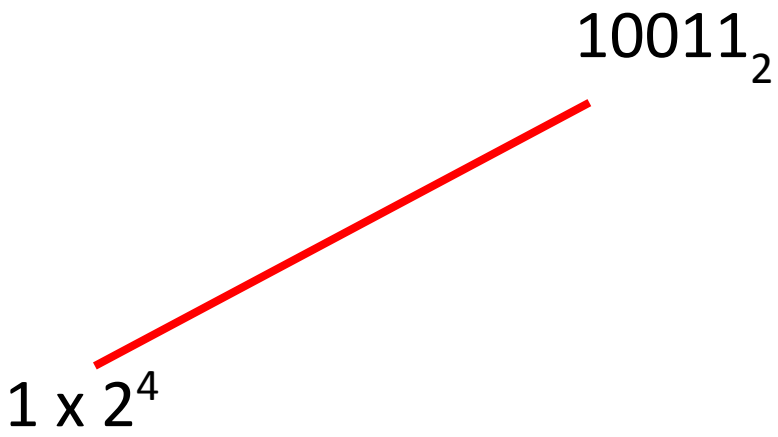


Binary to Decimal Example

10011_2

Binary to Decimal Example



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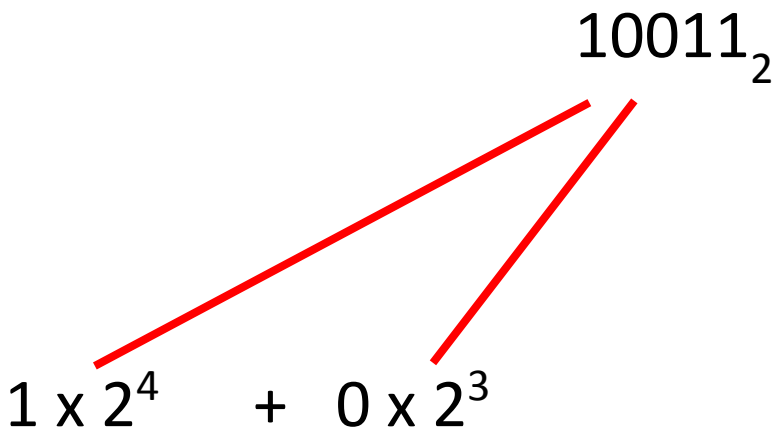
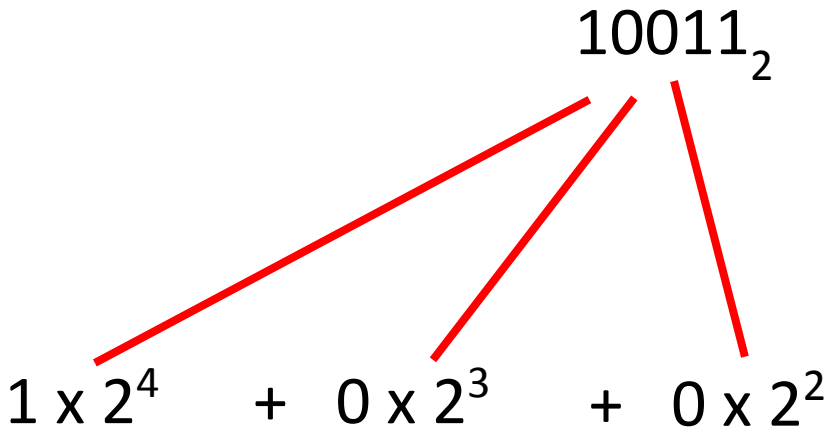
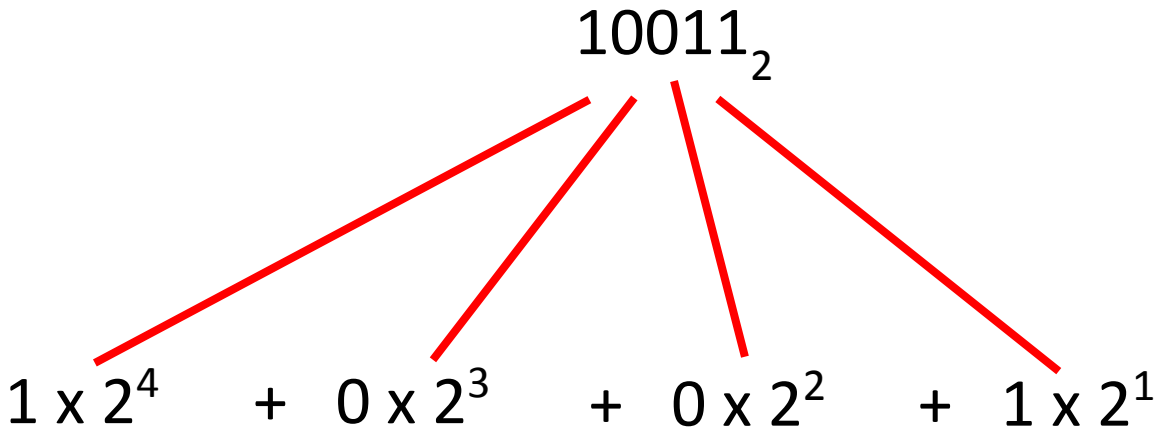


Binary to Decimal Example



Binary to Decimal Example



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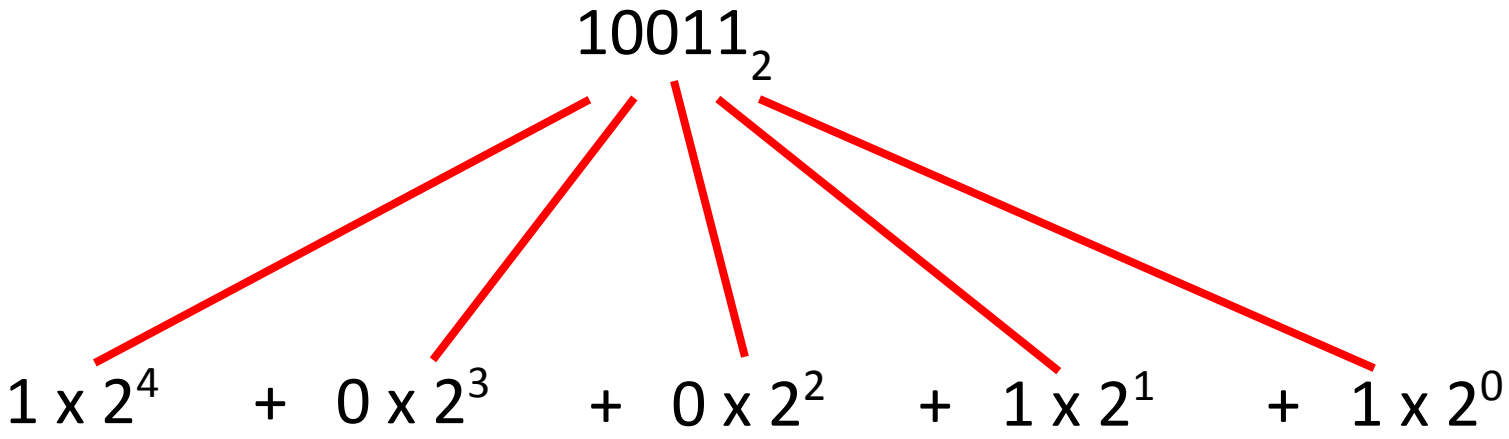
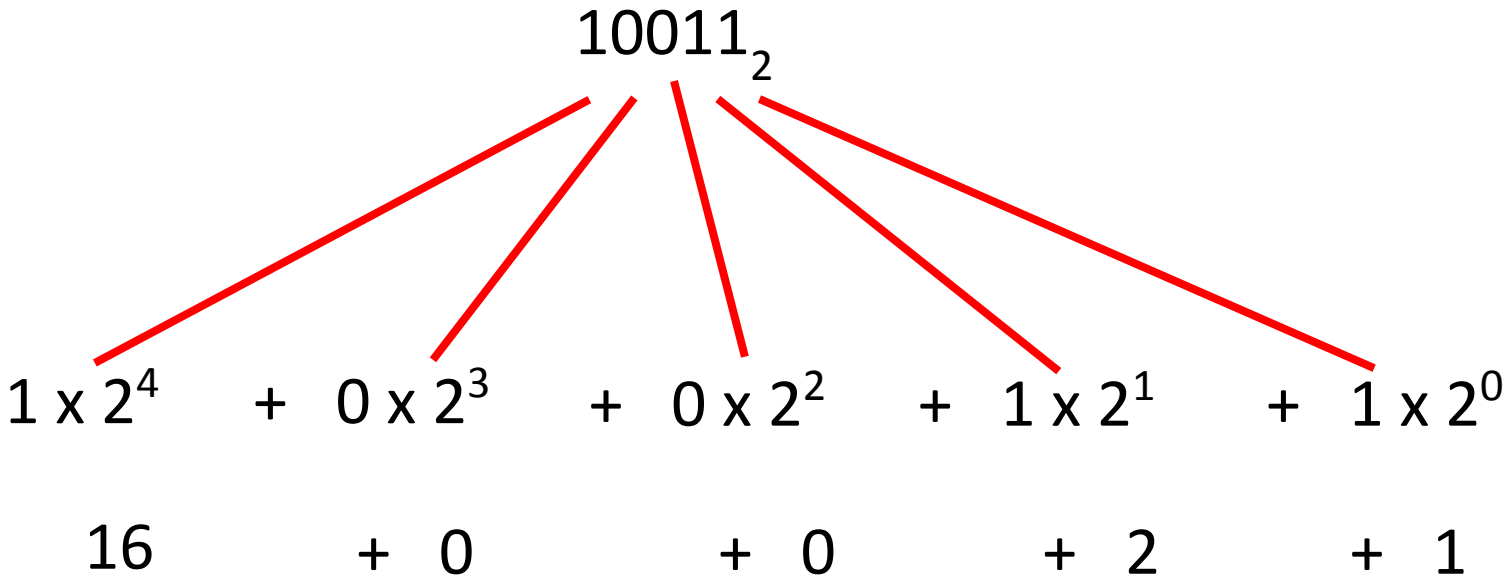
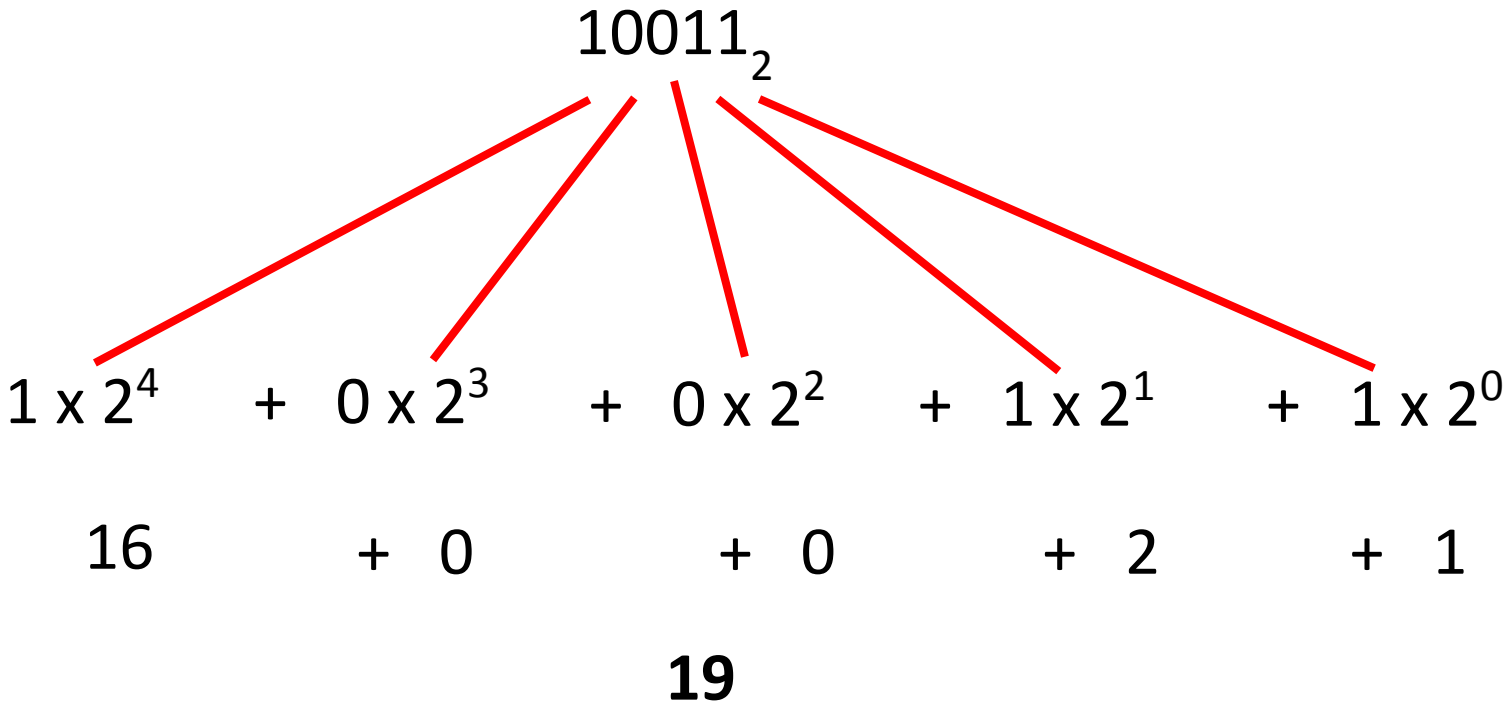


Binary to Decimal Example

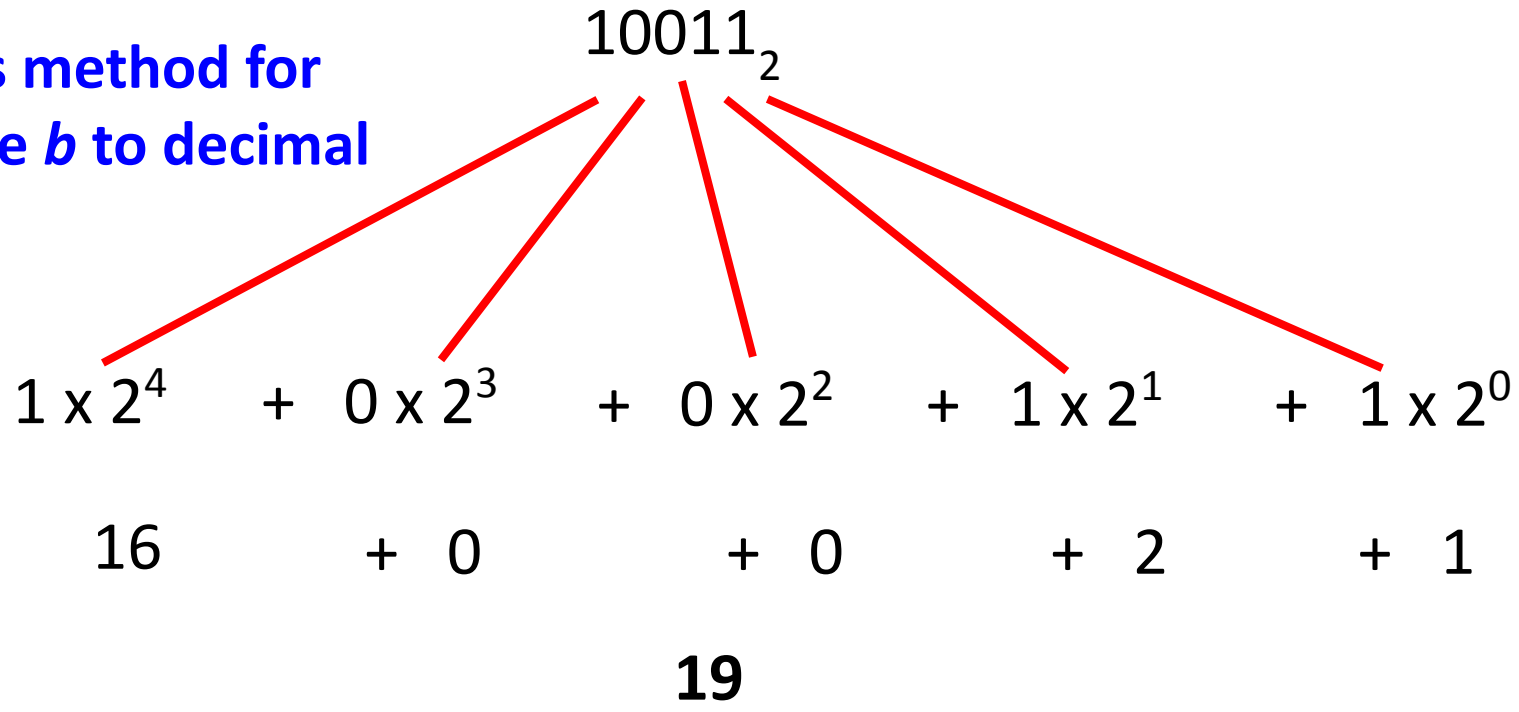


Binary to Decimal Example



Binary to Decimal Example

Use this method for any base *b* to decimal



Hexadecimal to Decimal Example

$2DF_{16}$

Hexadecimal to Decimal Example

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7

8	9	10	11	12	13	14	15
8	9	A	B	C	D	E	F

$2DF_{16}$

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$2DF_{16}$

2×16^2

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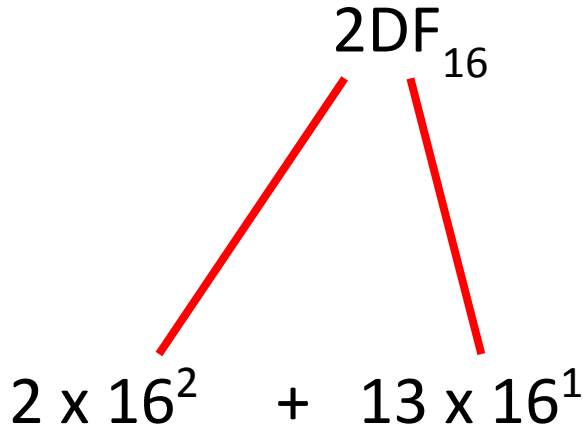

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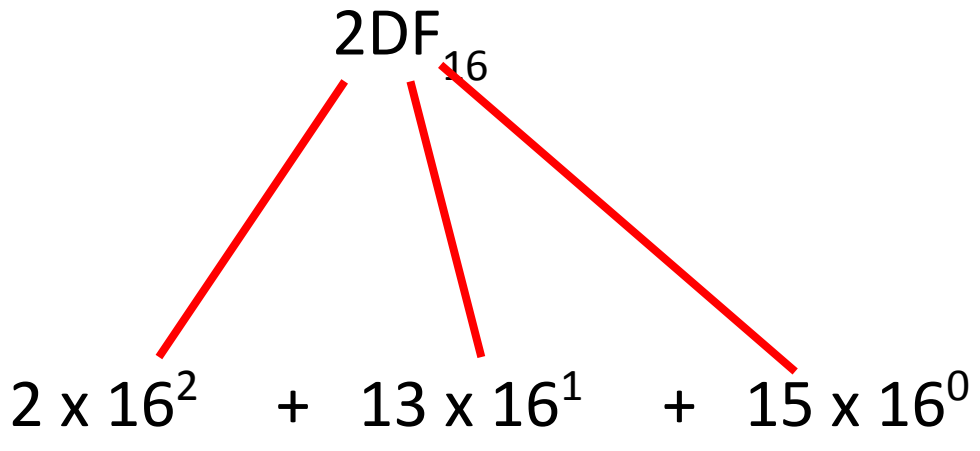

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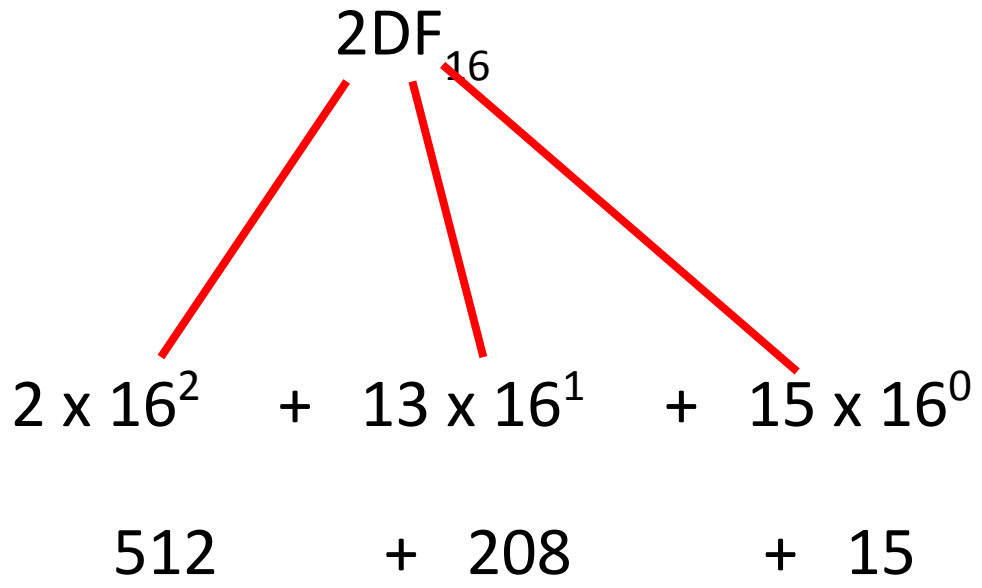

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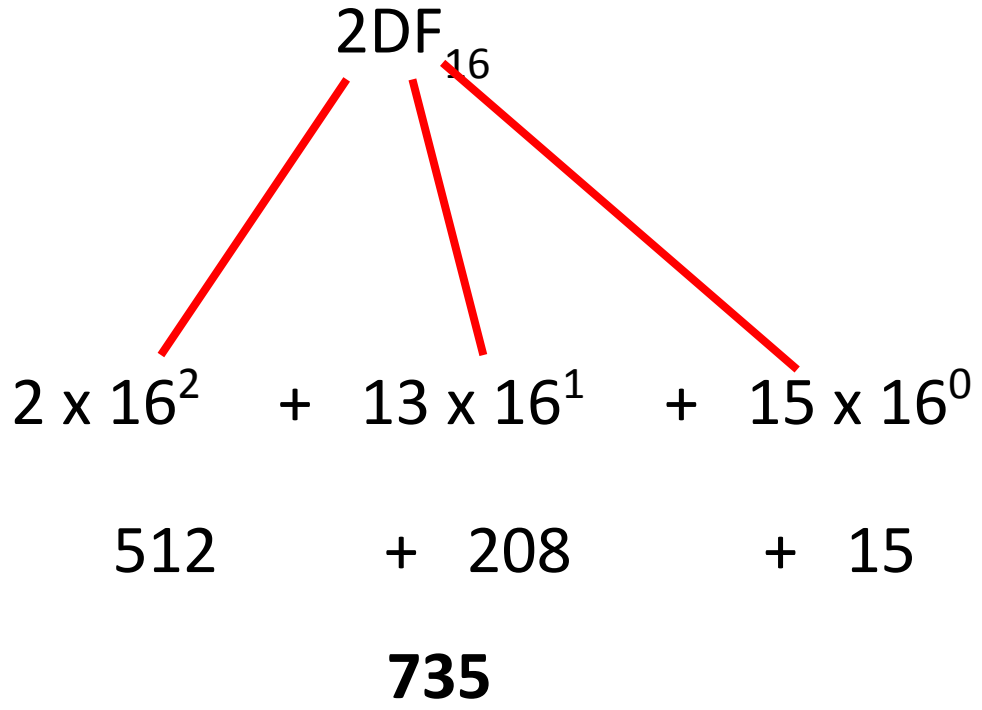

Binary to Hexadecimal Example

11010010_2

Binary to Hexadecimal Example

1 hex symbol can represent 4 bits (2^4)

$$11010010_2$$

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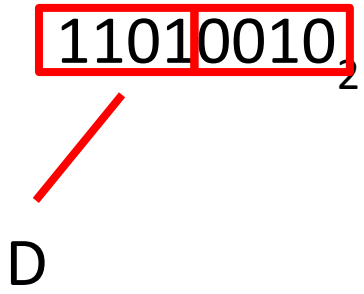

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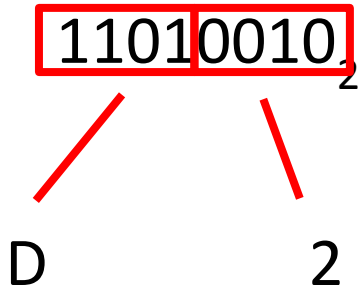

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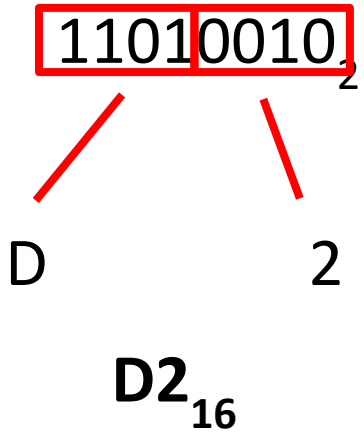

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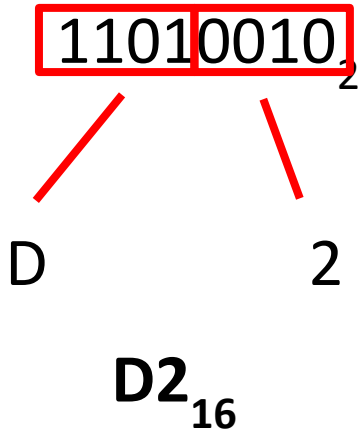

Hex to binary is no different

Properties of Binary Representation

- Even or odd?
  - Look at the LSB.

$1001011\mathbf{1}_2$ **Odd: 151**

Properties of Binary Representation

- Even or odd?
  - Look at the LSB.
- Bitwise shifts
  - Left = Multiply by 2.
  - Right = Divide by 2.

10010111_2 **Odd: 151**

100101110_2 **302**

001001011_2 **75**

Properties of Binary Representation

- Even or odd?
  - Look at the LSB.
- Bitwise shifts
  - Left = Multiply by 2.
  - Right = Divide by 2.
- Multiplying any two *n* bit binary values requires to most 2n bits.
  - Adding requires at most n+1 bits.

10010111_2 **Odd: 151**

100101110_2 **302**

001001011_2 **75**

Computing in Binary

- The smallest addressable unit of data on computer is a byte.
  - This is precisely 8 bits.
  - We can represent 2^8 (256) values.
- If we want to talk bits, we need to be a *bit* more clever.

```
// sum primes up to limit by sieving
long long sum_primes(long long limit){
   long long sum = 0;
   bool* marks = malloc(sizeof(bool)*(limit+1));
   if(marks == NULL)
   {
      perror("Malloc failed!");
      abort();
   }
   for(long long i=0; i < limit+1; i++){
      marks[i] = false;
   }
   for(long long i = 2; i <= limit; i++){

      if(!marks[i]){
         sum+=i;

         for(long long m=2*i; m <= limit; m += i)
         {
            marks[m] = true;
         }
      }
   }
   free(marks);
   return sum;
}
```

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```

7 bits are unused…

The Bitmap

Idea: Each bit of a byte represents a true or false value.

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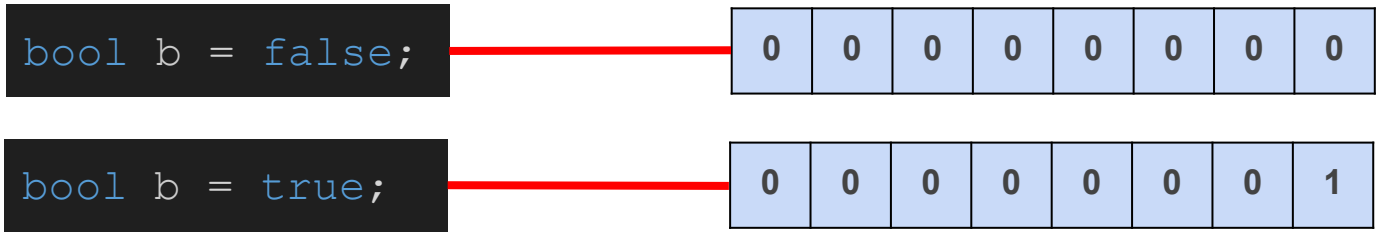

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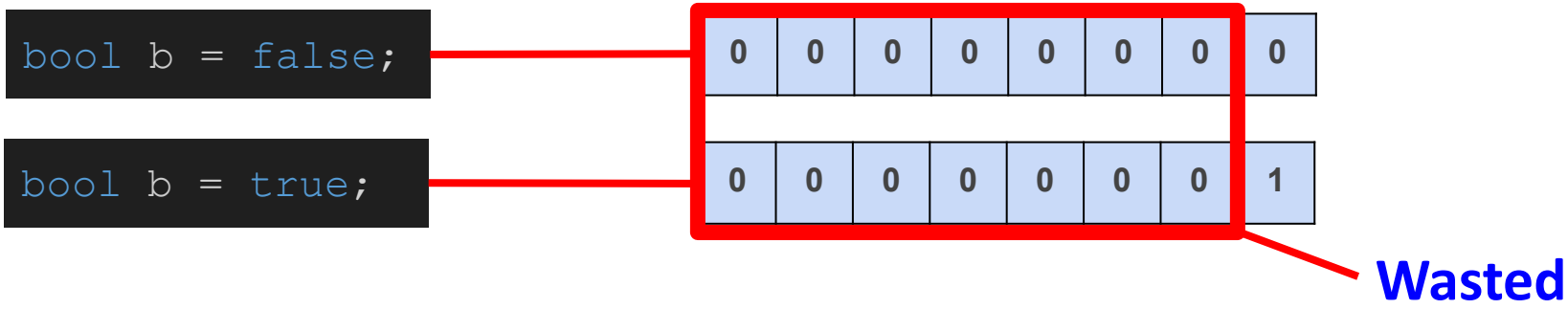

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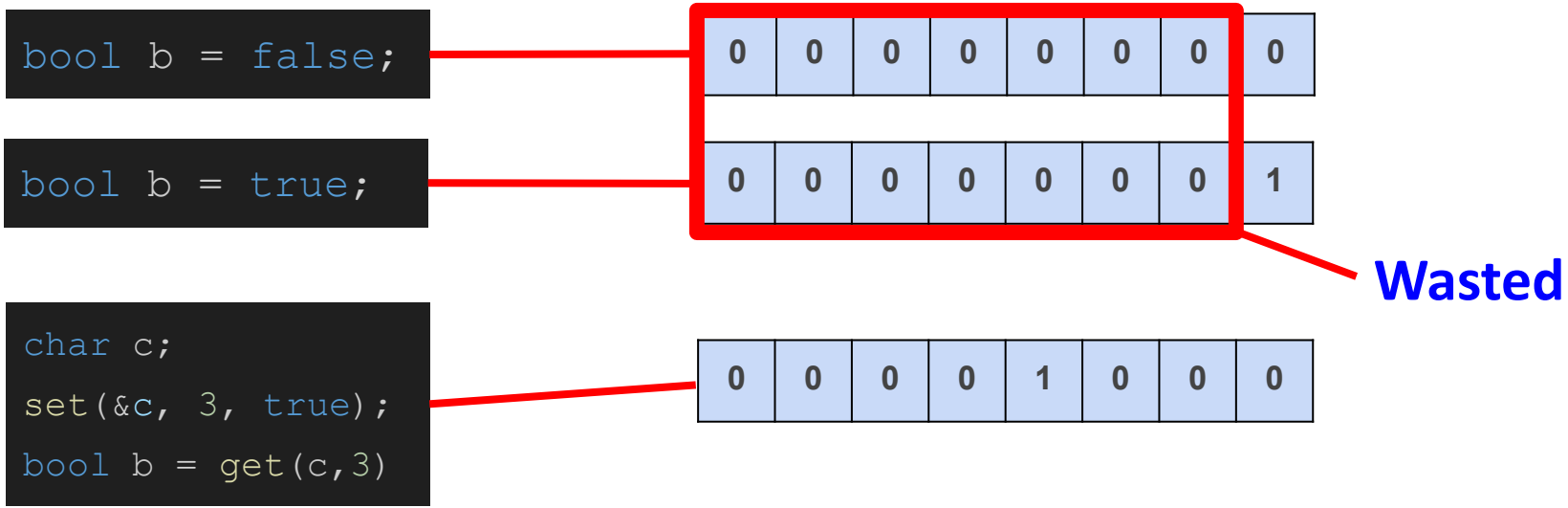

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