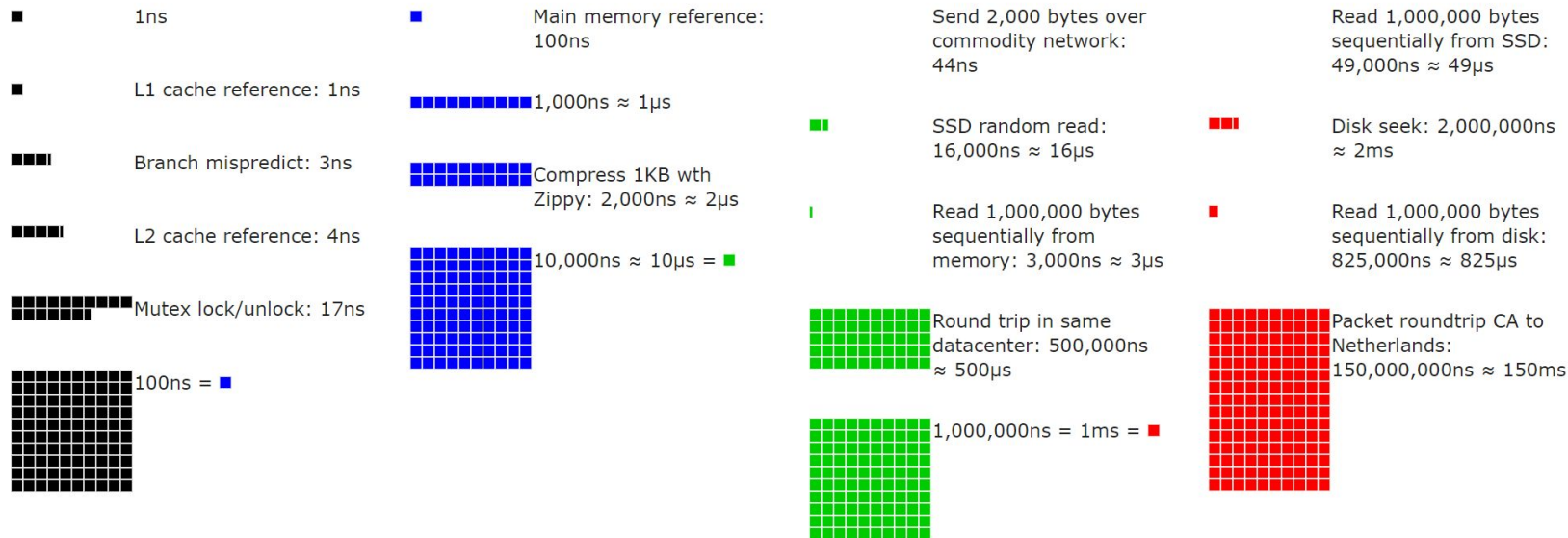


# Recitation 10

## Computer Architecture (section 1)

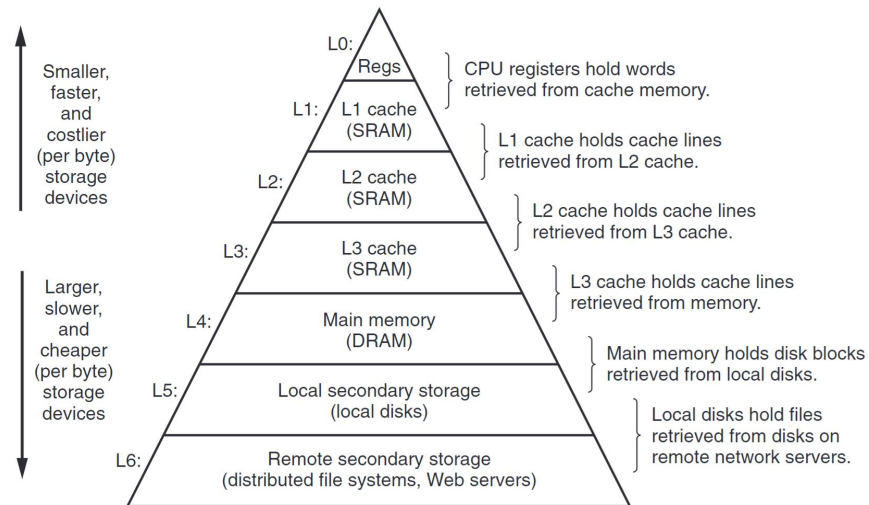
# Numbers every programmer should know



[https://colin-scott.github.io/personal\\_website/research/interactive\\_latency.html](https://colin-scott.github.io/personal_website/research/interactive_latency.html)

# Caches

- How can we hide memory access latency?
  - Exploit locality
    - Temporal
    - Spatial
- A cache is a fast memory store used for frequently accessed data.
  - Basically operates as a hash table.



# Cache Organization

$2^s$  Sets

Valid?	Tag ( $t$ bits)	Cache Block ( $2^b$ bytes)

Set 0

Set 1

Set 2

Set 3

$E$  lines  
per set

# Cache Organization

$2^s$  Sets

	Valid?	Tag ( $f$ bits)	Cache Block ( $2^b$ bytes)
Set 0			
Set 1			
Set 2			
Set 3			

$E$  lines  
per set

How do memory  
addresses map into  
the cache?

Address



# Cache Organization

$2^s$  Sets

	Valid?	Tag ( $t$ bits)	Cache Block ( $2^b$ bytes)
Set 0			
Set 1			
Set 2			
Set 3			

$E$  lines  
per set

How do memory addresses map into the cache?

Address



$$\text{Cache size} = 2^b \times E \times 2^s$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How many sets are in this cache?



# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How many sets are in this cache?

$$\text{Cache Size} = 2^b \times E \times 2^s$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How many sets are in this cache?

$$\text{Cache Size} = 2^b \times E \times 2^s$$

$$2048 = 64 \times 4 \times 2^s$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How many sets are in this cache?

$$\text{Cache Size} = 2^b \times E \times 2^s$$

$$2048 = 64 \times 4 \times 2^s$$

$$2^s = \mathbf{8 \text{ sets}}$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How does a 32-bit memory address map into this cache?

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How does a 32-bit memory address map into this cache?

$$2^b=64, 2^s=8$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How does a 32-bit memory address map into this cache?

$$2^b=64, 2^s=8$$

$$b=6, s=3, t = 32-(6+3)$$

# Practice Problem

You are given a system with the following configuration:

- 32-bit address space
- Cache size of 2048 bytes
- Block size of 64 bytes
- 4 lines per set

How does a 32-bit memory address map into this cache?

$$2^b=64, 2^s=8$$

$$b=6, s=3, t = 32-(6+3)$$

Tag (23 bits)	Set Index (3 bits)	Block Offset (6 bits)
------------------	-----------------------	--------------------------

# Cache Mapping Techniques

By varying the number of lines per cache set ( $E$ ), the associativity of the cache changes.



# Cache Mapping Techniques

By varying the number of lines per cache set ( $E$ ), the associativity of the cache changes.

- 1 line per set
  - A direct mapped cache

# Cache Mapping Techniques

By varying the number of lines per cache set ( $E$ ), the associativity of the cache changes.

- 1 line per set
  - A direct mapped cache
- 2,3,4... (n) lines per set
  - n-way set-associative cache

# Cache Mapping Techniques

By varying the number of lines per cache set ( $E$ ), the associativity of the cache changes.

- 1 line per set
  - A direct mapped cache
- 2,3,4... (n) lines per set
  - n-way set-associative cache
- All lines in a single set
  - Fully-associative cache

# Direct Mapped Cache

	Valid?	Tag	Cache Block (8 bytes)	
Set 0				<b><math>E = 1</math> line per set</b>
Set 1				
Set 2				
Set 3				
Set 4				
Set 5				
Set 6				
Set 7				

# Direct Mapped Cache

	Valid?	Tag	Cache Block (8 bytes)	
Set 0				$E = 1$ line per set
Set 1				
Set 2				
Set 3				
Set 4				
Set 5				
Set 6				
Set 7				

How do memory addresses map into the cache?

# Direct Mapped Cache

	Valid?	Tag	Cache Block (8 bytes)
Set 0			
Set 1			
Set 2			
Set 3			
Set 4			
Set 5			
Set 6			
Set 7			

$E = 1$   
line  
per set

How do memory addresses map into the cache?

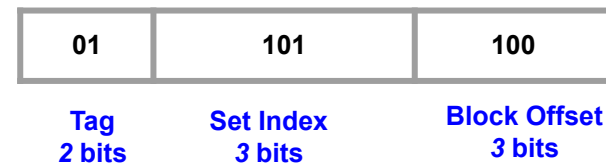
## 8-bit Address



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	0		
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	0		
Set 6	0		
Set 7	0		

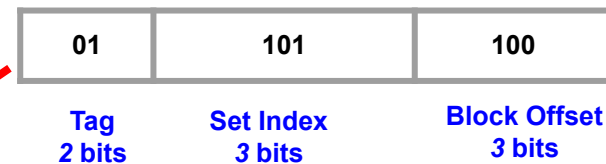
## Read Request



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	0		
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	0		
Set 6	0		
Set 7	0		

## Read Request



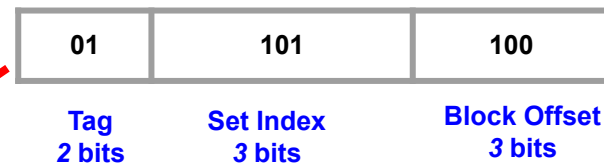
**Cache miss**



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	0		
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	0000▣000
Set 6	0		
Set 7	0		

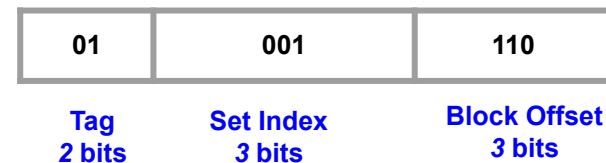
## Read Request



# Direct Mapped Cache in Action

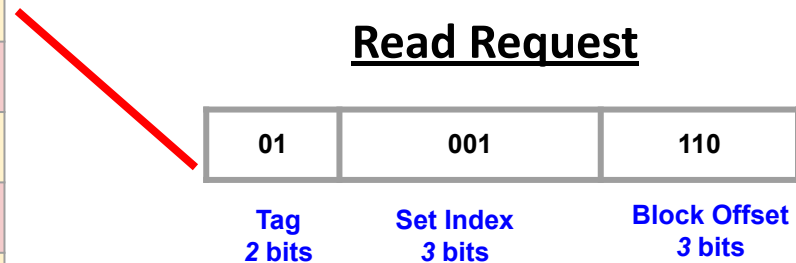
	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	0		
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	00000000
Set 6	0		
Set 7	0		

## Read Request



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	0		
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	00000000
Set 6	0		
Set 7	0		

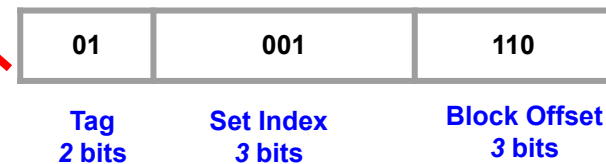


**Cache miss**

# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	1	01	00000000
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	00000000
Set 6	0		
Set 7	0		

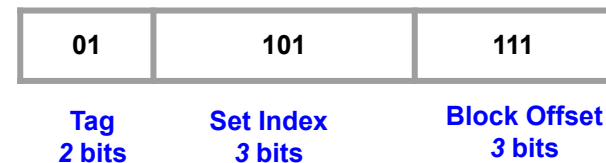
## Read Request



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	0		
<b>Set 1</b>	1	01	○○○○○○○○
<b>Set 2</b>	0		
<b>Set 3</b>	0		
<b>Set 4</b>	0		
<b>Set 5</b>	1	01	○○○○○○○○
<b>Set 6</b>	0		
<b>Set 7</b>	0		

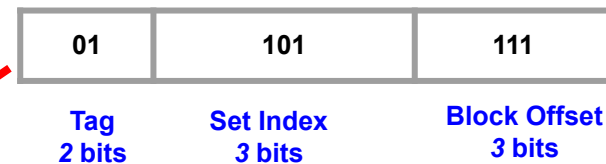
## Read Request



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	1	01	○○○○○○○○
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	■○○○○○○○
Set 6	0		
Set 7	0		

## Read Request

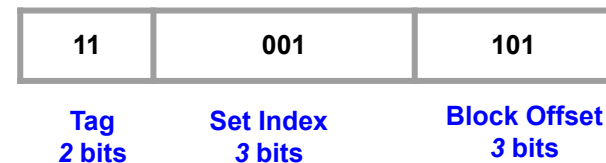


**Cache hit**

# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	0		
<b>Set 1</b>	1	01	○○○○○○○○
<b>Set 2</b>	0		
<b>Set 3</b>	0		
<b>Set 4</b>	0		
<b>Set 5</b>	1	01	○○○○○○○○
<b>Set 6</b>	0		
<b>Set 7</b>	0		

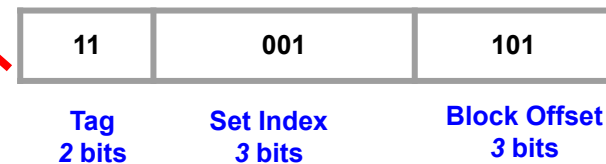
## Read Request



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	1	01	00000000
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	00000000
Set 6	0		
Set 7	0		

## Read Request



**Cache miss**

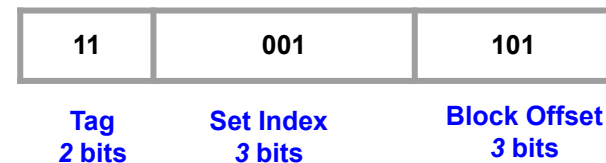
**Tags do not match!**



# Direct Mapped Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
Set 1	1	11	00000000
Set 2	0		
Set 3	0		
Set 4	0		
Set 5	1	01	00000000
Set 6	0		
Set 7	0		

## Read Request



# Set-associative Cache (4-way)

	Valid?	Tag	Cache Block (8 bytes)	
<b>Set 0</b>				<b><math>E = 4</math> lines per set</b>
<b>Set 1</b>				

# Set-associative Cache (4-way)

	Valid?	Tag	Cache Block (8 bytes)	
<b>Set 0</b>				<b><math>E = 4</math> lines per set</b>
<b>Set 1</b>				

How do memory addresses map into the cache?

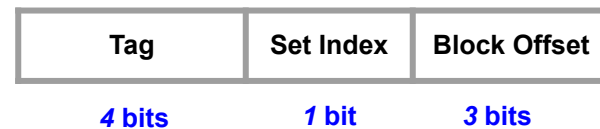
# Set-associative Cache (4-way)

	Valid?	Tag	Cache Block (8 bytes)
Set 0			
Set 1			

$E = 4$   
lines  
per set

How do memory addresses map into the cache?

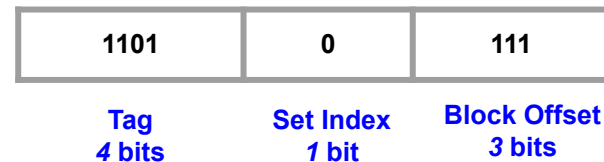
## 8-bit Address



# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	0		
	0		
	0		
	0		
<b>Set 1</b>	0		
	0		
	0		
	0		

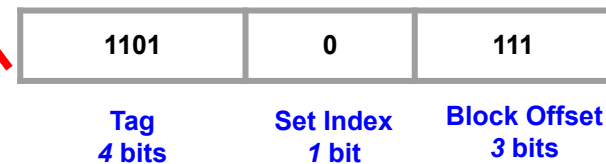
## Read Request



# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	0		
	0		
	0		
	0		
Set 1	0		
	0		
	0		
	0		

## Read Request

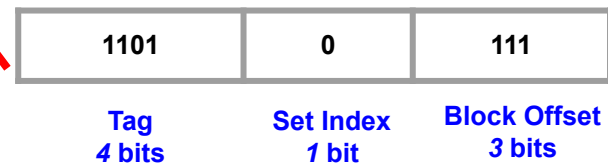


**Cache miss**

# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	☐0000000
	0		
	0		
	0		
<b>Set 1</b>	0		
	0		
	0		
	0		

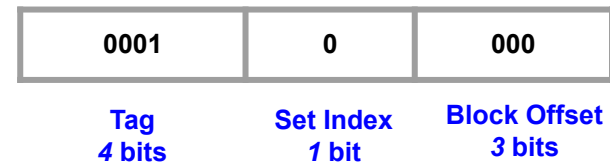
## Read Request



# 4-way Set Associative Cache in Action

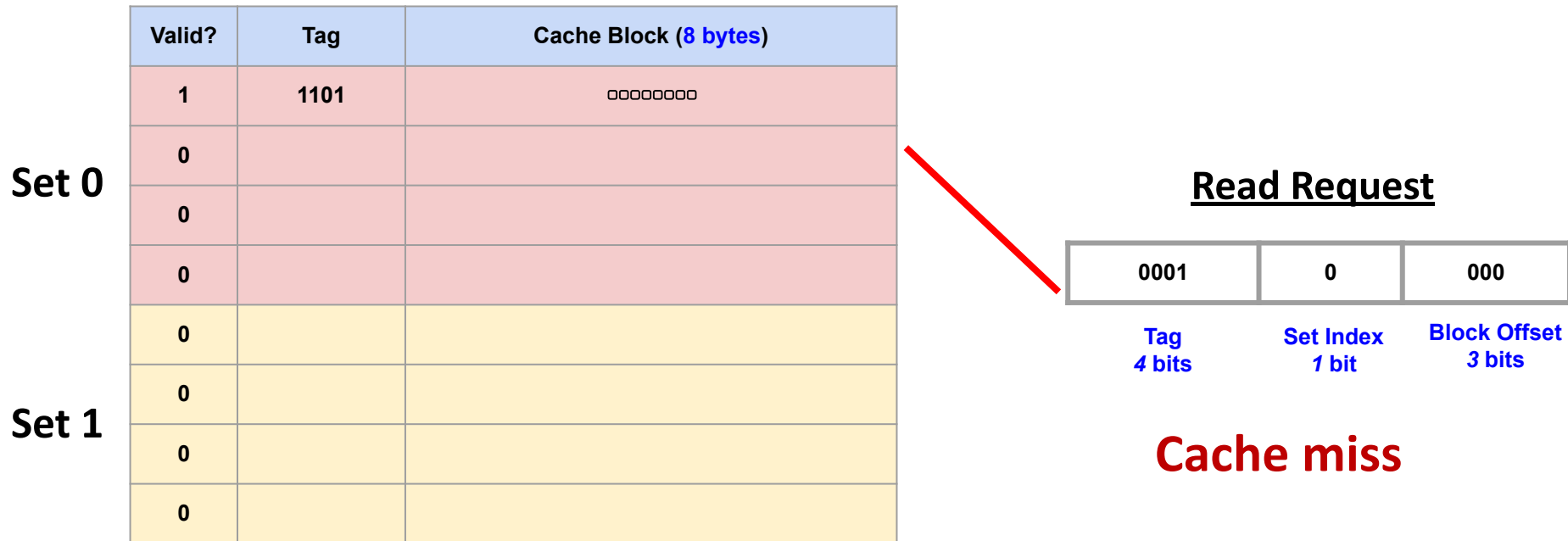
	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	□□□□□□□□
	0		
	0		
	0		
<b>Set 1</b>	0		
	0		
	0		
	0		

## Read Request

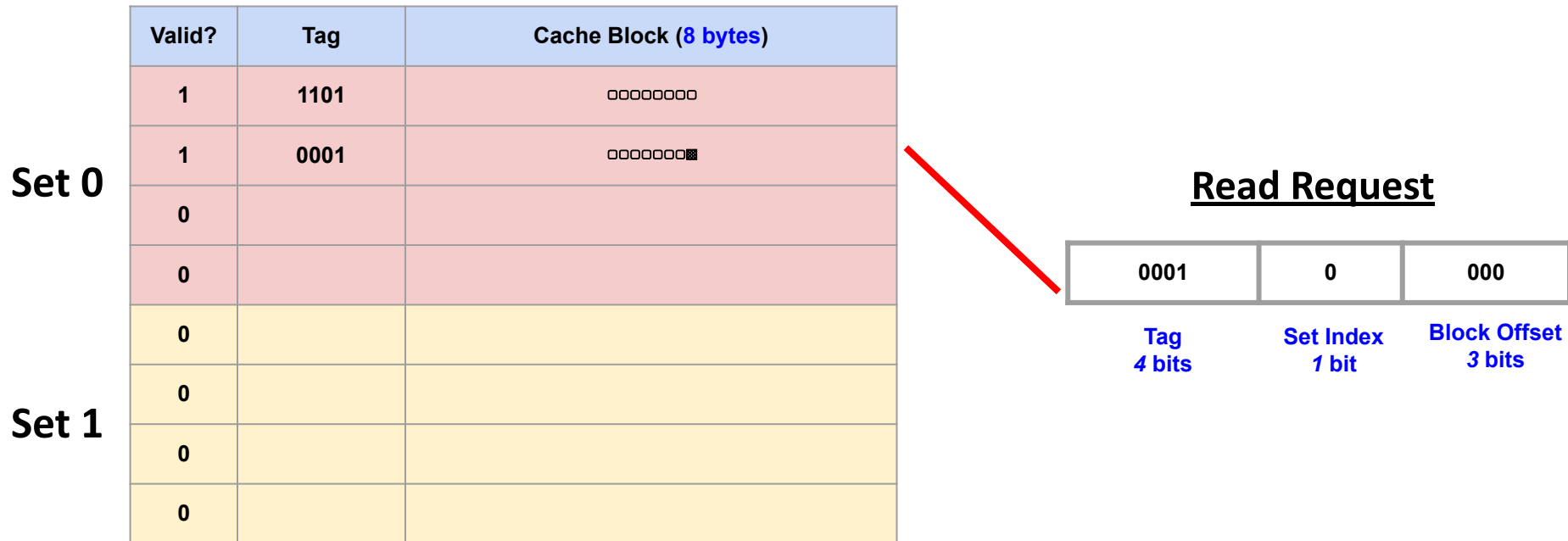




# 4-way Set Associative Cache in Action



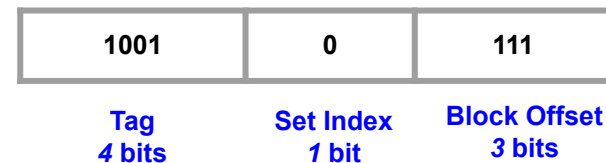
# 4-way Set Associative Cache in Action



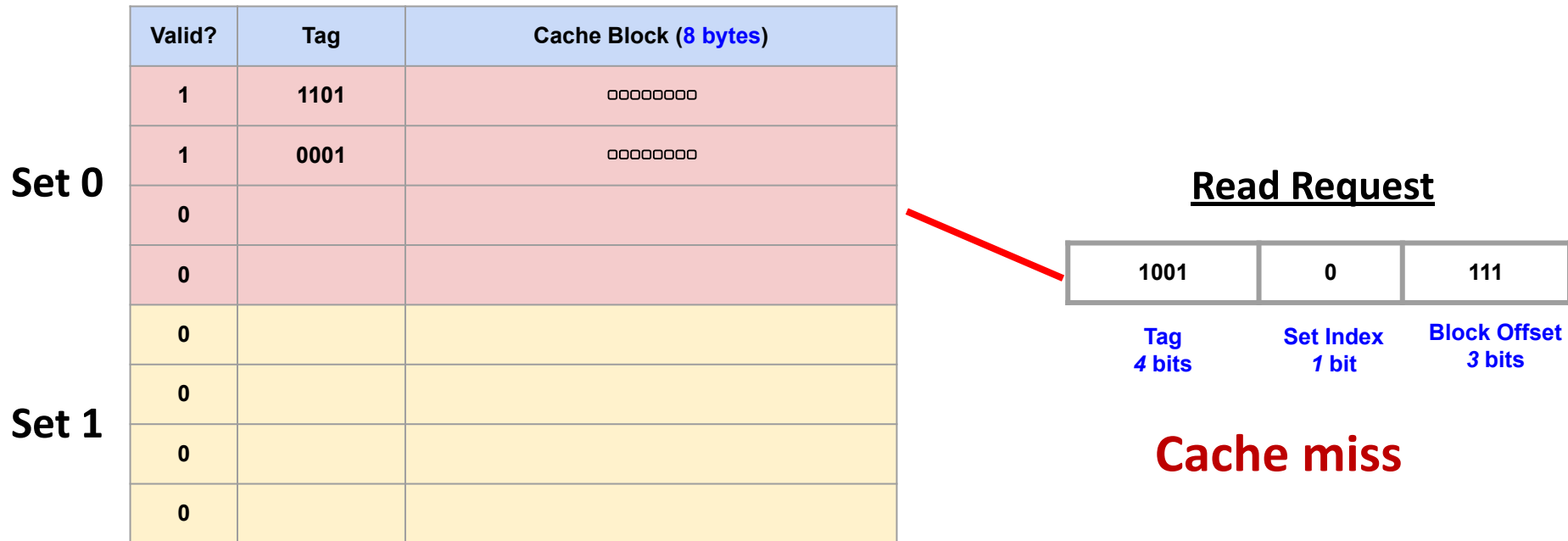
# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	○○○○○○○○
	1	0001	○○○○○○○○
	0		
	0		
<b>Set 1</b>	0		
	0		
	0		
	0		

## Read Request



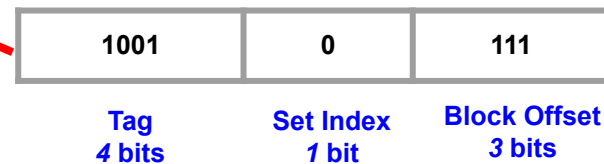
# 4-way Set Associative Cache in Action



# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	1	1101	00000000
	1	0001	00000000
	1	1001	☒00000000
	0		
Set 1	0		
	0		
	0		
	0		

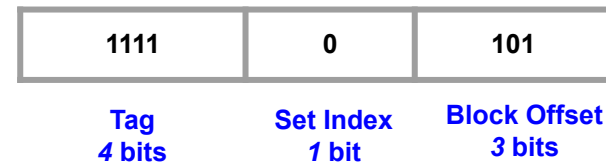
## Read Request



# 4-way Set Associative Cache in Action

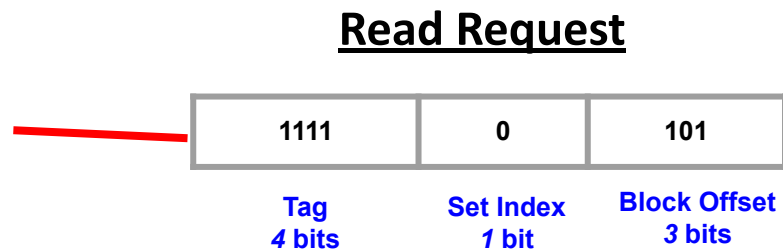
	Valid?	Tag	Cache Block (8 bytes)
Set 0	1	1101	○○○○○○○○
	1	0001	○○○○○○○○
	1	1001	○○○○○○○○
	0		
Set 1	0		
	0		
	0		
	0		

## Read Request



# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	1	1101	○○○○○○○○
	1	0001	○○○○○○○○
	1	1001	○○○○○○○○
	0		
Set 1	0		
	0		
	0		
	0		

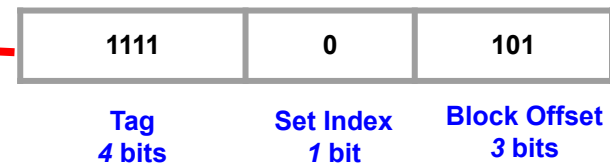


**Cache miss**

# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	○○○○○○○○
	1	0001	○○○○○○○○
	1	1001	○○○○○○○○
	1	1111	○○○○○○○○
<b>Set 1</b>	0		
	0		
	0		
	0		

## Read Request

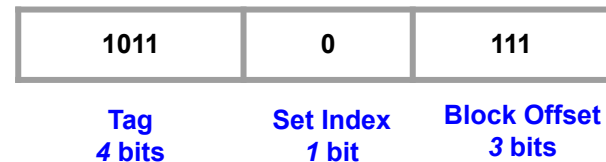




# 4-way Set Associative Cache in Action

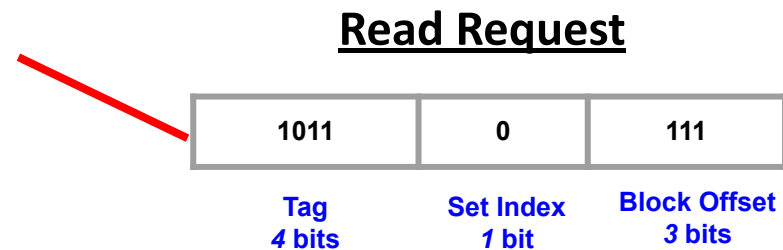
	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
	1	0001	00000000
	1	1001	00000000
	1	1111	00000000
<b>Set 1</b>	0		
	0		
	0		
	0		

## Read Request



# 4-way Set Associative Cache in Action

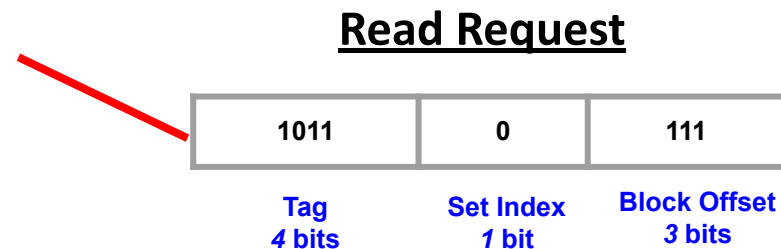
	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
	1	0001	00000000
	1	1001	00000000
	1	1111	00000000
<b>Set 1</b>	0		
	0		
	0		
	0		



**Cache miss**

# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	oooooooo
	1	0001	oooooooo
	1	1001	oooooooo
	1	1111	oooooooo
<b>Set 1</b>	0		
	0		
	0		
	0		



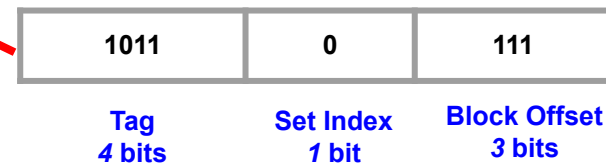
**Cache miss**

**We need to evict a line  
from the cache.**

# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
Set 0	1	1101	oooooooo
	1	0001	oooooooo
	1	1001	oooooooo
	1	1111	oooooooo
Set 1	0		
	0		
	0		
	0		

## Read Request

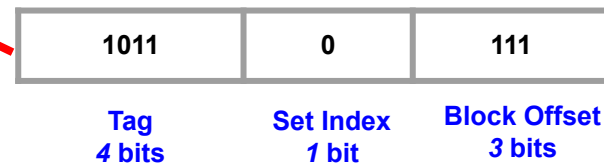


To evict, we use a cache eviction policy. Let's evict at random for now.

# 4-way Set Associative Cache in Action

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
	1	0001	00000000
	1	1011	☒00000000
	1	1111	00000000
<b>Set 1</b>	0		
	0		
	0		
	0		

## Read Request



# Fully Associative Cache

	Valid?	Tag	Cache Block (8 bytes)
Set 0			

$E = 8$   
lines  
per set

# Fully Associative Cache

Set 0

Valid?	Tag	Cache Block (8 bytes)

$E = 8$   
lines  
per set

How do memory addresses map into the cache?

# Fully Associative Cache

**Set 0**

Valid?	Tag	Cache Block (8 bytes)

*E* = 8 lines per set

How do memory addresses map into the cache?

## 8-bit Address





# Fully Associative Cache

Set 0

Valid?	Tag	Cache Block (8 bytes)

$E = 8$   
lines  
per set

How do memory addresses map into the cache?

8-bit Address



With 1 set we don't need a set index

# Writing to the Cache

So far we have seen cache behaviour for memory reads.

How should the cache behave when we write to memory?

# Writing to the Cache

So far we have seen cache behaviour for memory reads.

How should the cache behave when we write to memory?

*Should we write directly to memory and ignore the cache?*

*Should we write to both the cache and memory?*

*Should we only write to the cache?*

# Write-through Cache

If memory block is in cache:

1. Write changes to cache.
2. Then write changes to memory.

# Write-through Cache

If memory block is in cache:

1. Write changes to cache.
2. Then write changes to memory.

**Set 0**

**Set 1**

Valid?	Tag	Cache Block (8 bytes)
1	1101	□□□□□□□□
1	1001	□□□□□□□□

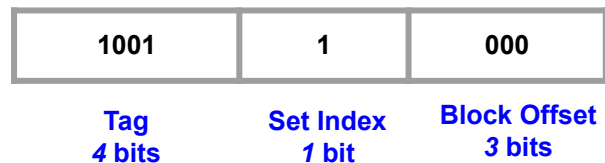
# Write-through Cache

If memory block is in cache:

1. Write changes to cache.
2. Then write changes to memory.

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
<b>Set 1</b>	1	1001	00000000

## Write Request



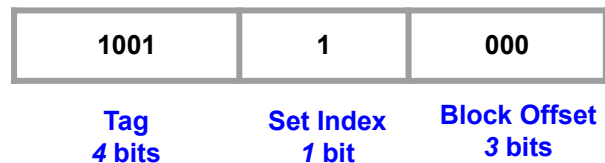
# Write-through Cache

If memory block is in cache:

1. Write changes to cache.
2. Then write changes to memory.

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
<b>Set 1</b>	1	1001	00000000

## Write Request



# Write-through Cache

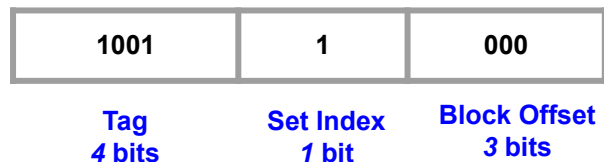
If memory block is in cache:

1. Write changes to cache.
2. Then write changes to memory.

**Now we write this line to memory.**

	Valid?	Tag	Cache Block (8 bytes)
Set 0	1	1101	00000000
Set 1	1	1001	00000000

**Write Request**





# Write-back Cache

If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

# Write-back Cache

If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

# Write-back Cache

If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	0	oooooooo
<b>Set 1</b>	1	1001	0	oooooooo

# Write-back Cache

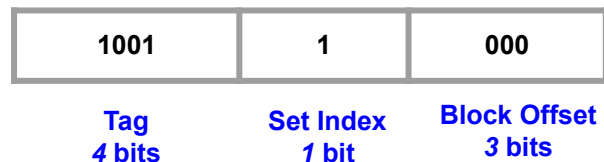
If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	0	○○○○○○○○
<b>Set 1</b>	1	1001	0	○○○○○○○○

## Write Request



# Write-back Cache

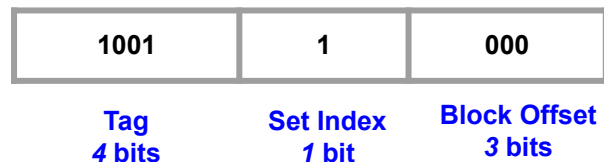
If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
Set 0	1	1101	0	00000000
Set 1	1	1001	1	00000001

## Write Request



# Write-back Cache

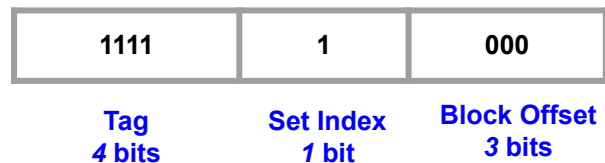
If memory block is in cache:

1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	0	00000000
<b>Set 1</b>	1	1001	1	00000000

## Read Request



# Write-back Cache

If memory block is in cache:

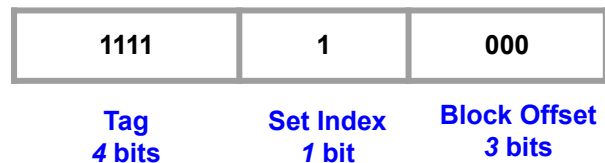
1. Write changes to cache.
2. On eviction of the cache line, write changes to memory.

The caveat is we must now store a *dirty bit* for each line.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
Set 0	1	1101	0	oooooooo
Set 1	1	1001	1	oooooooo

**Write to  
memory  
on  
eviction**

## Read Request



**Cache miss**

# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.



# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	0	1101	0	oooooooo
<b>Set 1</b>	0	1111	0	oooooooo

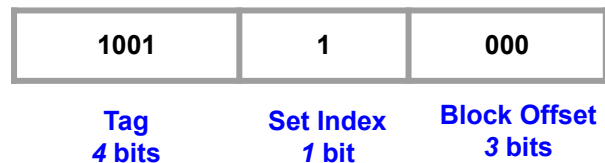
# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	0	1101	0	oooooooo
<b>Set 1</b>	0	1111	0	oooooooo

## Write Request



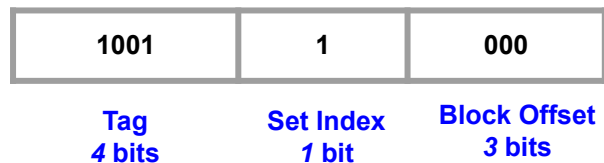
# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	0	1101	0	○○○○○○○○
<b>Set 1</b>	1	1001	0	○○○○○○○○

## Write Request



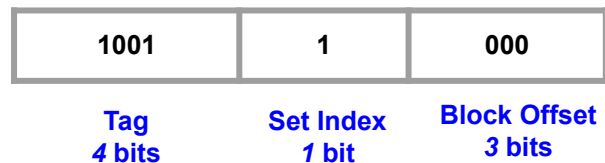
# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	0	1101	0	00000000
<b>Set 1</b>	1	1001	1	00000001

## Write Request



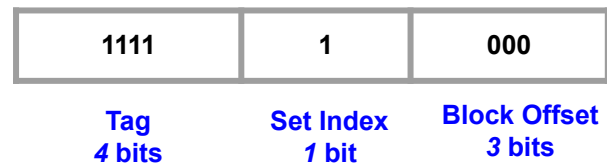
# Write-allocate Cache

If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
<b>Set 0</b>	0	1101	0	○○○○○○○○
<b>Set 1</b>	1	1001	1	○○○○○○○○

## Read Request



# Write-allocate Cache

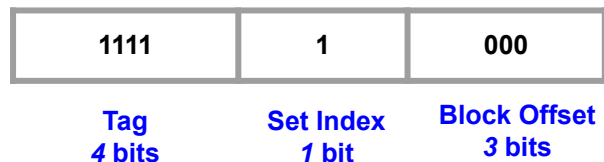
If memory block is not in cache:

1. Read block from memory into cache.
2. Use a write-back policy.

	Valid?	Tag	Dirty	Cache Block (8 bytes)
Set 0	0	1101	0	oooooooo
Set 1	1	1001	1	oooooooo

**Write to  
memory  
on  
eviction**

Read Request



**Cache miss**

# No-write-allocate Cache

If memory block is not in cache:

1. Directly write to memory.

With this policy, the cache is only populated on a read miss

# No-write-allocate Cache

If memory block is not in cache:

1. Directly write to memory.

With this policy, the cache is only populated on a read miss

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	○○○○○○○○
<b>Set 1</b>	1	1001	○○○○○○○○



# No-write-allocate Cache

If memory block is not in cache:

1. Directly write to memory.

With this policy, the cache is only populated on a read miss

	Valid?	Tag	Cache Block (8 bytes)
<b>Set 0</b>	1	1101	00000000
<b>Set 1</b>	1	1001	00000000

## Write Request

